MIND, BRAIN, AND EDUCATION

# Teaching by Analogy: From Theory to Practice

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**ABSTRACT**— Analogy is a powerful tool for fostering conceptual understanding and transfer in STEM and other fields. Well-constructed analogical comparisons focus attention on the causal-relational structure of STEM concepts, and provide a powerful capability to draw inferences based on a well-understood source domain that can be applied to a novel target domain. However, analogy must be applied with consideration to students' prior knowledge and cognitive resources. We briefly review theoretical and empirical support for incorporating analogy into education, and recommend five general principles to guide its application so as to maximize the potential benefits. For analogies to be effective, instructors should use well-understood source analogs and explain correspondences fully; use visuospatial and verbal supports to emphasize shared structure among analogs; discuss the alignment between semantic and formal representations; reduce extraneous cognitive load imposed by analogical comparison; and encourage generation of inferences when students have some proficiency with the material. These principles can be applied flexibly to topics in a wide variety of domains.

Our aim in this paper is to outline a general strategy for teaching by analogy, based on theoretical principles derived from extensive research on analogical problem solving. The overarching theme is that learning by analogy needs to be guided by the pragmatic goals of the learner (and of the teacher), and hence must focus on the *causal* structure of situations. Moreover, fostering transfer of learning with analogical reasoning requires attention to the varying cognitive resources that students possess.

Two situations are generally said to be analogous if they share a common pattern of relationships—typically including causal relations—among their constituent elements, even though the elements themselves differ across the two situations. Figure 1 illustrates the analogy between a restaurant kitchen and a biological cell (often used in biology classrooms). In general, one situation—the *source* analog (here the kitchen)—is more familiar, concrete, and/or comprehensible; the source serves to illuminate the *target* analog (here the cell), which is typically more novel and less well understood. Within each analog, various relations connect two or more entities, each of which plays a particular role in one or more relation. For example, in the source shown in Figure 1, *cooks prepare dishes*, and in the target *ribosomes produce proteins*. Analogical reasoning requires the identification of similar relations in the source and target (e.g., *prepare* and *produce*; see Lu, Wu, & Holyoak, 2019). Based on corresponding relations, it is possible to find a *mapping*: a set of systematic correspondences between entities in the source and target that are similar by virtue of their role patterns (e.g., *cooks* ➔ *ribosomes*, *dishes* ➔ *proteins*; see Gentner, 1983; Halford, Wilson, & Phillips, 1998; Corral & Jones, 2014). Importantly, the mapping hinges on similarity of relational roles, rather than direct similarities between individual entities (e.g., cooks do not resemble ribosomes). The key to learning from an instructional analogy is to first understand *why* the source behaves as it does, and to then transfer this knowledge to the target.

As is the case for many analogies that may be used in a classroom, the kitchen/cell analogy is not perfect. For example, restaurant kitchens do not reproduce by splitting in two, and the analogy does not capture the complex processes that regulate which genes are expressed in the nucleus. However, if these shortcomings are recognized by the instructor, the analogy can still be valuable. It provides a purposefully simplified representation of the complex behavior of a cell, and emphasizes the most important relations while backgrounding irrelevant information. In general, it is challenging (and often impossible) to find a source analog that perfectly instantiates every relation in the target concept and that introduces no extraneous information. But imperfect source analogs can still be useful in classroom teaching.

Many concepts taught in formal and practical disciplines (including but not restricted to the so-called STEM fields

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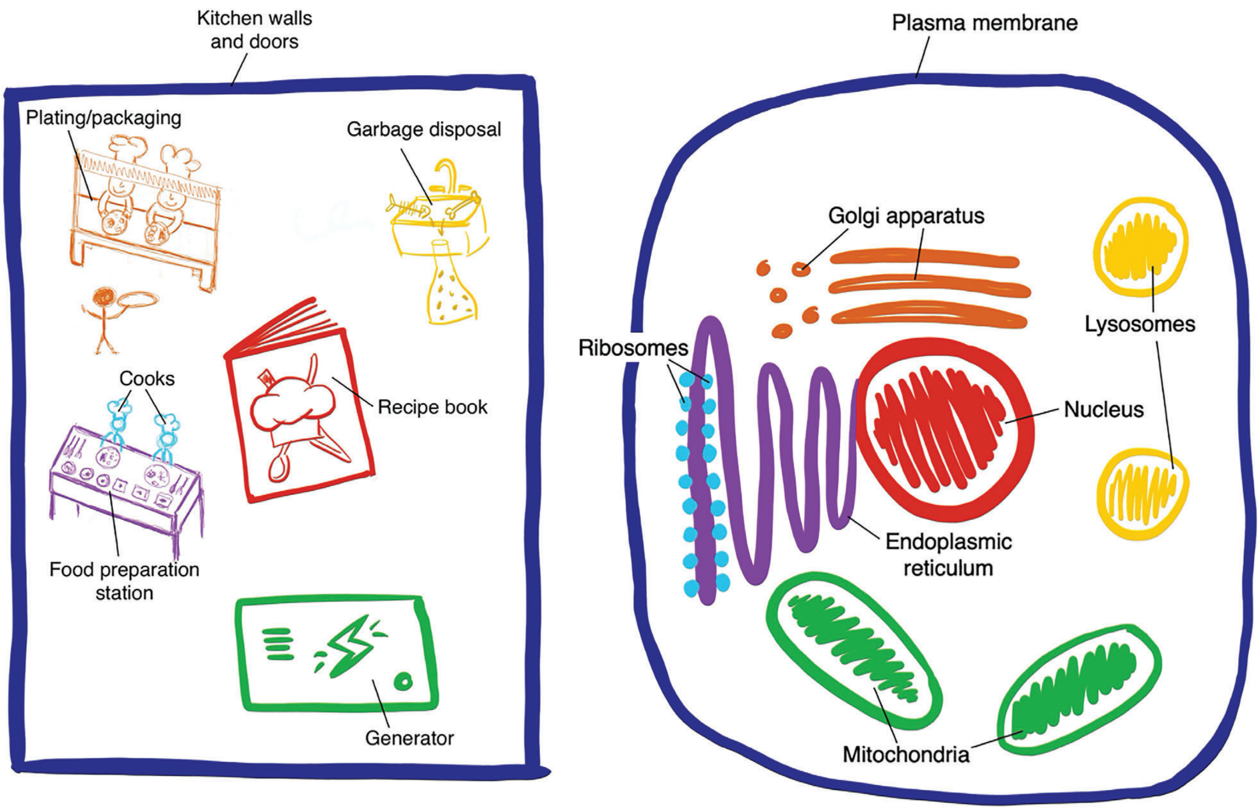


**Fig 1.** A visual representation of the analogy between a restaurant kitchen (source) and a cell (target). Corresponding elements are drawn in the same color. Drawing by Caryn Gray.

comprised science, technology, engineering, and mathematics) are relational in nature (English & Halford, 1995). Indeed, mathematics is in essence a system of patterns and relations (Devlin, 2012). It has been argued that the end goal of education is to foster the development of abstract relational schemas that can be applied flexibly in diverse situations (Goldwater & Schalk, 2016). For example, if a student learns how to analyze the structure of an argument in a philosophy class, but later fails to apply that knowledge to political arguments, their education has in an important sense failed. An analogy can provide a means of acquiring a relational schema, and may be particularly helpful when the target domain cannot be directly perceived—perhaps because it is too small (submicroscopic particles), too large (plate tectonics), or too abstract (the human mind). A well-conceived analogy has the ability to relate difficult-to-visualize constructs to a more tangible, imageable realm. Metaphors, although not always based directly on analogical reasoning, serve similar functions (Holyoak, 2019; Holyoak & Stamenković, 2018).

## ANALOGICAL PROBLEM SOLVING

Modern work on the use of analogy to solve problems and learn causal structure began with studies by Gick and Holyoak (1980, 1983). These investigators asked college students to role-play solving a medical problem (drawn from Gestalt psychology; Duncker, 1945) in which a doctor is faced with a patient suffering from a malignant stomach tumor. The tumor is inoperable, and the patient will die unless it is destroyed. There is a kind of ray that will destroy the tumor if the rays reach it at a high intensity. But unfortunately, such high-intensity rays will also destroy the healthy tissue they pass through on the way to the tumor. At lower intensities the rays will not damage the healthy tissue, but neither will they remove the tumor. How can the doctor use rays to destroy the tumor, while at the same time sparing the healthy tissue?

Gick and Holyoak provided some of their participants with a story in advance of the tumor problem. The story was introduced in the context of what was ostensibly a different task—people were asked to memorize it or write a brief summary. One version of the story, called "The General", described how a general captured a fortress located in the middle of a county by dividing his army into small groups and dispatching each group simultaneously down a different road (where the roads, like spokes on a wheel, all led to the fortress). Viewed as an educational intervention, it is important to note that the General story was not an actual situation with which participants were previously acquainted. Rather,

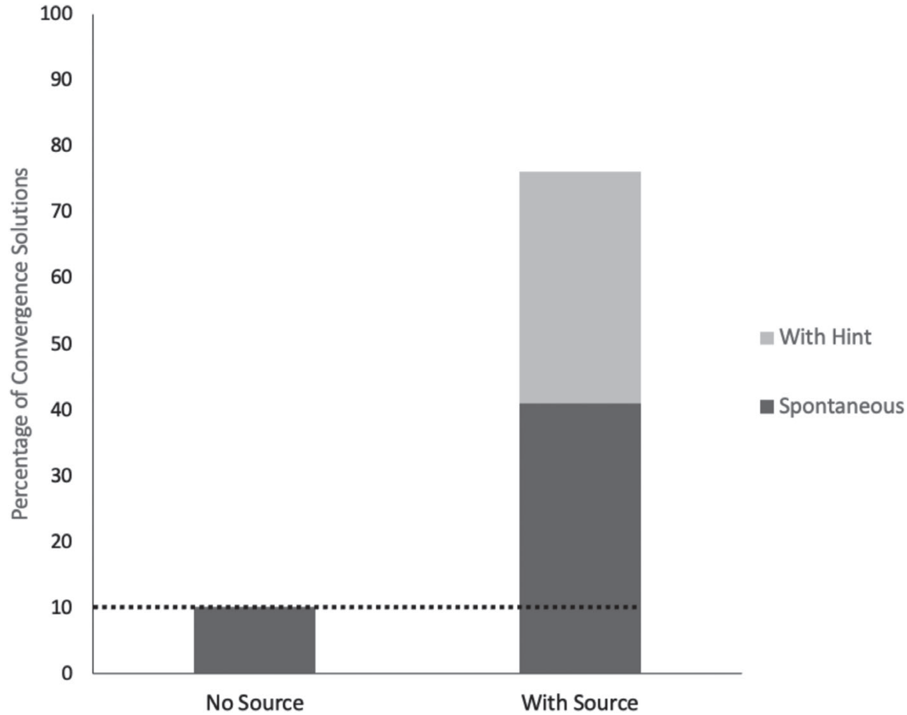


**Fig 2.** Frequency of producing the convergence solution to the tumor problem, either without any source analog or after seeing the General story (a potential source analog). Data from Gick and Holyoak (1980, Experiment V).

it was deliberately constructed by the experimenters to provide a close analogical match to the tumor problem. Just as the general employed small converging groups of soldiers to cross safely to the fortress and capture it, the doctor could use several ray machines positioned around the patient to direct multiple low-intensity rays at the tumor, simultaneously. The key idea is that each beam will pass harmlessly through healthy tissue, but the converging weak rays will summate at the focal point of the tumor, destroying it.

It is not easy to come up with this "convergence" solution (which is actually quite similar to standard medical practice for radiation therapy) without the aid of an analogy. As shown in Figure 2, only about 10% of the students generated this solution in the absence of the General story (dotted horizontal line). When the students read the story just prior to working on the tumor problem, roughly 30 % *more* of them generated the convergence problem spontaneously. That is, these students noticed that the General story was relevant and used it to solve the problem. When the experimenters followed up with a simple cue that "the story you read earlier might give some hints," another 30 % succeeded in finding the convergence solution. The overall picture we obtain from these experiments is of a glass half full or half empty—a substantial proportion of people spontaneously used the source analog to solve the problem, though about as many initially failed to notice its relevance (yet often succeeded later when given a hint).

The basic findings from Gick and Holyoak (1980, 1983), coupled with later extensions of this work (Catrambone & Holyoak, 1989; Holyoak, Junn, & Billman, 1984; Holyoak & Koh, 1987; Keane, 1988), provide a theoretical framework for understanding how analogies can be used in education. As illustrated in Figure 3, analogical problem solving involves several major components (Holyoak, Novick, & Melz, 1994). Once the source analog has been *accessed* (either spontaneously or after a cue to consider it), key elements of the two analogs can be *mapped* to generate correspondences among the elements. Critically, these correspondences are largely based on matches between relations that are in some sense *causal* (cf. Holyoak, Lee, & Lu, 2010; Schank & Abelson, 1977)—functionally relevant to the dynamic changes that occur within each analog. The general and the doctor have similar *goals*, which provide *reasons* for their choices of actions; in each situation states or actions (dividing the troops, finding multiple ray machines) *enable* or *prevent* subsequent actions, which *cause* state changes (capturing the fortress, destroying the tumor).

Once a mapping has been established between source and target, the problem solver can *infer* a novel solution to the target problem by creating a new idea ("apply converging weak rays") based on information initially provided by the source alone ("send small groups down many roads"). Finally, in the aftermath of exploring the analogy it is possible to *learn* something new, by forming a more abstract

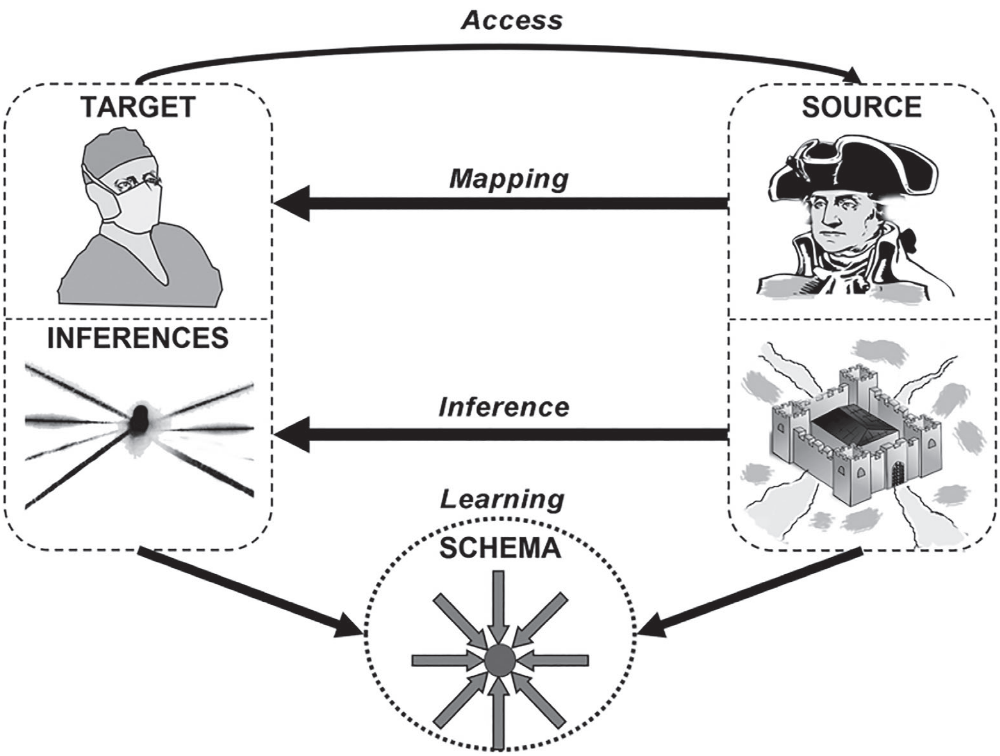


**Fig 3.** Major processes involved in reasoning and learning by analogy, illustrated using the General story and the tumor problem from Gick and Holyoak (1980, 1983). Reprinted by permission from Holyoak (2019).

relational schema that captures the commonalities between the source and target: roughly, when a large force cannot be safely applied to a centrally located object, apply multiple converging small forces instead. Repeated examples that fit a common analogical schema will set up a positive feedback loop—people find it easier and easier to spontaneously apply the schema to additional examples (Catrambone & Holyoak, 1989; Gentner, Loewenstein, & Thompson, 2003). Schema induction (Gick & Holyoak, 1983) is the basic mechanism by which analogy can foster flexible transfer and generalization of knowledge. Critically with respect to its role in education, analogy can be used as device to teach the causal structure of novel problems not only for adults, but also for children as young as preschool age (Brown, Kane, & Echols, 1986; Holyoak et al., 1984).

It is important to emphasize the central role of causal relations (as opposed to arbitrary relations unrelated to goals or important outcomes) in the educational use of analogies. Consider the analogy between water waves and sound waves (the earliest major scientific analogy, dating from the era of imperial Rome). The concrete source analog of water waves provides a deeper understanding of sound. Sound is analogous to water waves in that sound exhibits a pattern of behavior corresponding to that of water waves: propagating across space with diminishing intensity, passing around small barriers, rebounding off large barriers, and so on. Perceptual features and non-causal relations are very different (water makes objects wet upon contact, air does not), but the underlying pattern of causal relations among the elements is similar.

In this example, like most analogies involving empirical phenomena, the key functional relations involve causes and their effects. By transferring knowledge about causal relations, the analogy provides a new explanation of why various phenomena occur. In general, analogical thinking is guided by the pragmatic goals of the reasoner (Holland, Holyoak, Nisbett, & Thagard, 1989; Holyoak, 1985; Holyoak & Thagard, 1989), which direct attention to relevant causal relations (Spellman & Holyoak, 1996). For example, people are more likely to base analogical inferences on the relation "A causes B" than on the formally comparable but not non-causal relation "A occurs before B" (Lassaline, 1996). Moreover, humans have specialized mechanisms for reasoning about cause-effect relations (for a review see Holyoak & Cheng, 2011), and these directly impact analogical transfer. In particular, people naturally distinguish between generative causes (which tend to make something happen) and preventive causes (which tend to stop something from happening), and treat the two types of causes differently when drawing analogical inferences (Holyoak, Lee, & Lu, 2010; Lee & Holyoak, 2008; for a review see Holyoak & Lee, 2017).

## OBSTACLES TO SUCCESSFUL TEACHING BY ANALOGY

Although a comprehensive review of the benefits of incorporating analogical reasoning in STEM education is beyond the scope of this article, positive impacts on learning outcomes have been robustly demonstrated (see Alfieri, Nokes-Malach, & Schunn, 2013, for a more thorough review). For example, analogical comparison of physics concepts increases far transfer (Nokes-Malach, VanLehn, Belenky, Lichtenstein, & Cox, 2013), and several studies have shown that including analogies in science texts increases comprehension of the scientific material in question and its causal relational structure (Braasch & Goldman, 2010; Clement & Yanowitz, 2003; Jaeger & Wiley, 2015). In the domain of mathematics, it has been shown that analogical comparison of worked examples illustrating proportionality improves performance on test problems and reduces common misconceptions (Begolli & Richland, 2016).

While the learning outcomes associated with including analogies in learning opportunities are typically positive, the approach is not without its potential pitfalls. Students in any classroom are not uniform. They do not come to a learning opportunity with the same expectations, prior knowledge, or cognitive resources, and this variability in student populations undoubtedly affects the efficacy of any proposed educational interventions. Previous research has implicated executive functions (e.g., working memory and inhibitory control) in analogical reasoning (Cho et al., 2010; Cho, Holyoak, & Cannon, 2007; Gray & Holyoak, 2020; Hummel & Holyoak, 1997, 2003; Kubricht, Lu, & Holyoak, 2017; Tohill & Holyoak, 2000; Waltz, Lau, Grewal, & Holyoak, 2000) and in academic achievement (Campos, Almeida, Ferreira, Martinez, & Ramalho, 2013). Students with fewer cognitive resources may be less able to benefit from analogies in educational settings. In fact, some research suggests that less able students may even be harmed by the presence of analogies in educational material (Jaeger & Wiley, 2015; Zook & Maier, 1994).

The difficulty inherent in learning relational concepts can be compounded by ineffective teaching techniques that fail to highlight the structural nature of STEM concepts. Compared to K-12 instructors in Japan and Hong Kong, American mathematics instructors do not effectively highlight the conceptual structure of mathematics and connections between mathematics concepts (Richland, Zur, & Holyoak, 2007). Given that for semantically distant analogies, novices often do not notice relational similarity spontaneously (Gick & Holyoak, 1980), they may fail to attend to the causal structure that defines STEM concepts and relations among them without instructional guidance.

Because executive functions are correlated significantly with academic achievement (Campos et al., 2013), students with fewer cognitive resources at their disposal are already at a significant disadvantage in most educational settings. The learning capabilities of these students should not be discounted, however, as some studies have shown that children who receive low scores on figural analogy tests have similar potential for learning as children with higher scores (Touw, Vogelaar, Verdel, Bakker, & Resing, 2017). Furthermore, if insufficient executive functioning is the root of problem, educational analogical interventions can be designed to lessen the cognitive load on the reasoner (e.g., Begolli & Richland, 2016; Richland et al., 2007; Richland & McDonough, 2010).

## AN ANALOGICAL APPROACH TO TEACHING: FIVE PRINCIPLES

Here we sketch an analogical approach to teaching based on a set of principles drawn from research in cognitive psychology and cognitive neuroscience. The principles are summarized in Table 1. The analogical approach provides a theoretically motivated and empirically supported framework within which instructors can devise their own specific interventions to maximize the utility of analogy in education. The current recommendations build upon previous guides to analogical teaching (see, for example Treagust, Harrison, & Venville, 1998; Vendetti, Matlen, Richland, & Bunge, 2015). Here we consolidate techniques that serve the same general instructional goals, propose additional methods to incorporate analogy into classrooms, and highlight the importance of considering limitations in students' cognitive resources. The principles we describe here are not intended to constitute an exhaustive set, but they all serve to highlight causal relations that are crucial to learning and transfer, particularly in STEM fields.

One of the strengths of the analogical approach is its flexibility. It is not a prescription for a specific educational intervention, or a call for instructors to drastically alter their teaching style. Rather, the analogical approach provides a set of principles to guide instructors as they make changes (perhaps quite modest) to the way in which they present material to their students. Here we will review each principle and an example of its application separately, but the components should be applied in tandem whenever appropriate.

### Capitalize on Prior Knowledge

Analogies allow learners to use prior knowledge to better understand an unfamiliar topic. Teaching by analogy is useful when an imbalance in knowledge exists, and the reasoner can draw on prior knowledge of a source domain to aid in understanding an unfamiliar target domain. The positive effect of relevant prior knowledge on learning is well documented (Chiesi, Spilich, & Voss, 1979; McNamara &

**Table 1**
Summary of Principles for Analogical Approach to Teaching

1. Use well-understood source analogs to capitalize on prior knowledge. Explain correspondences fully.
2. Highlight shared causal structure among examples of a structurally defined category using visuospatial, gestural, and verbal supports.
3. Explain correspondences between semantic information and mathematical operations. Discuss conceptual meaning of mathematical operations.
4. Use presentation style to facilitate comparison and reduce cognitive load of comparison process when appropriate.
5. Once students have some proficiency with the material, encourage generation of inferences.

Kintsch, 1996). In general, prior knowledge alters the encoding of new knowledge (Boshuizen & Schmidt, 2008; Gobet & Simon, 1996; Kimball & Holyoak, 2000). Grasping that some new material is analogous to something already known allows the reasoner to fit corresponding elements of the target material into roles and relations that are already stored in memory, thereby aiding comprehension (Bean, Searles, Singer, & Cowen, 1990). The powerful ability to draw analogical inferences can guide scientific discovery and improve understanding (Holyoak & Thagard, 1995; Yanowitz, 2001). Further, using a familiar real-world experience as a source analog may increase student motivation, as the analogy provides an example of the application and relevance of classroom content (Duit, 1991). Effective source analogs are not only familiar but also relatively concrete, a property associated with richer and more distributed neural representations (Binder, 2016).

Reviews of science textbooks show that many textbook authors are sensitive to this characteristic of analogy. Approximately 90% of analogies found in a collection of college-level biochemistry textbooks related concrete source analogs to abstract target concepts (Orgill & Bodner, 2006). Several studies have used analogies in efforts to improve comprehension of a difficult-to-visualize concept (Baker & Lawson, 2001; Braasch & Goldman, 2010; Jaeger, Taylor, & Wiley, 2016). One study investigated the use of analogy to aid understanding of El Niño weather systems (Jaeger et al., 2016). Participants with low spatial reasoning abilities (i.e., those who would likely have the most trouble visualizing the large-scale, hard-to-visualize weather system) showed significant improvement in comprehension when they received a text drawing an analogy to a small scale, more easily imageable situation (letting the air out of a balloon). The benefit of analogy has also been demonstrated in classroom settings. College-level genetics students showed increased comprehension of abstract genetics concepts after receiving instructional analogies in comparison to students in the same course who did not receive analogies (Baker & Lawson, 2001).

Although using analogy to capitalize on relevant prior knowledge can be helpful in many circumstances, instructors should keep several considerations in mind when applying this component of analogy to improve instruction. First, a student must possess the requisite prior knowledge in order to make use of it. Several studies have demonstrated that analogies are particularly helpful for individuals with a high level of prior knowledge (Braasch & Goldman, 2010; Jee et al., 2013; Rittle-Johnson, Star, & Durkin, 2009). Educators should not assume that all students have sufficient prior knowledge to benefit from an analogy discussed in class. This is especially true for classrooms that include students of mixed backgrounds, as what might seem to be common knowledge for a "typical" American-born student may not be common knowledge for all students.

While it is not always easy to assess the prior knowledge of each student in a classroom, some strategies can address imbalance of prior knowledge. If time allows, prior knowledge can be gauged with pretests, which may contribute to better learning outcomes in their own right (Richland, Kornell, & Kao, 2009). In addition, the educator can ensure that all students possess a basic understanding of the source by explaining the source analog in full. In so doing, the educator can explicitly highlight the relevant (and salient but irrelevant) components of the source, informing students with low prior knowledge about important aspects of the source domain. Although research indicates that analogy is particularly beneficial for learners with high prior knowledge of the source (Braasch & Goldman, 2010; Jee et al., 2013), the "threshold" that must be met in order to benefit from analogical instruction may be fairly low (Rittle-Johnson et al., 2009).

In addition, if multiple source analogs are available, it may be useful to have students compare them to identify their commonalities (Gick & Holyoak, 1983; Loewenstein, Thompson, & Gentner, 2003). Recent work on teaching adolescents about climate change and complex systems suggests that presenting two source analogs may increase understanding of the source material and support greater transfer if the comparison among the analogs is sufficiently scaffolded (Jacobson et al., 2020).

In addition, instructors should not assume that students know how to effectively utilize analogies, even if they have been identified by an instructor (Venville, Bryer, & Treagust, 1994). Simply stating a cell is analogous to a restaurant kitchen may leave some students uncertain about which aspects of the kitchen are like the cell and which aspects are not. To address this limitation, educators should take care to explain the source analog and its correspondences to the target concept fully. Another consideration is that novices may spontaneously focus on the target concept (because their goal as a learner is to understand it). However, mapping tends to be more accurate when it proceeds *from* the more coherent and well-understood source analog *to* the less well-understood target (Kubose, Holyoak, & Hummel, 2002). To facilitate successful use of prior knowledge, it is advisable to direct students' attention first to the source analog.

Finally, in nearly all cases the analogy will not be perfect. Some aspects of the source analog may not have corresponding elements in the target concept, and thus should not be carried over to the target concept. In order to prevent inappropriate prior knowledge from producing misconceptions, instructors should explicitly map the relevant correspondences and point out the limits of the analogy—that is, which aspects of the source analog are irrelevant. When possible, the description of the source may be selectively tailored to optimize the analogical match with the target.

A study by Bean et al. (1990) illustrates effective use of prior knowledge in analogical teaching. In this study, an analogy was utilized to introduce high school students to the structure and function of a cell and its parts. In the experimental analogy condition, researchers used a factory to model the difficult-to-visualize interrelationships among the components of a cell. The experimental condition utilized some components of the analogical approach. This condition included a pictorial illustration, a study guide indicating key correspondences, and explicit instruction in the mappings. For example, students were instructed that the cell's mitochondria corresponded to the factory's power plant, because both served the same function of providing energy to the system. This instruction effectively draws upon relevant prior knowledge, establishes the corresponding elements of the source and target, and emphasizes the functional relational similarities that define the analogy. On a subsequent test, students in the experimental condition outperformed a second group of students who did not receive the picture, a third group who received the study guide only, and a control group who read a textbook chapter about the cell and its functions.

**Highlight Shared Structure**

One of the most important uses of analogy in STEM education leverages alignment and analogical comparison to focus attention on shared relational information. If an instructor's goal is to emphasize the shared structure that defines a concept, analogous examples of the concept can be aligned and compared. Research has demonstrated that analogical comparison increases attention to relations (Catrambone & Holyoak, 1989; Gentner & Markman, 1997; Gick & Holyoak, 1983; Goldwater & Gentner, 2015; Kotovsky & Gentner, 1996), and that alignment and analogical comparison of exemplars improves learning outcomes in STEM fields (Alfieri et al., 2013; Begolli & Richland, 2016; Gentner et al., 2016; Klein, Piacente-Cimini, & Williams, 2007; Nokes-Malach et al., 2013; Richland & McDonough, 2010).

There is strong evidence that analogical reasoning is a resource-intensive process. Analogical processing relies on several constructs that comprise cognitive capacity, including fluid intelligence, working memory, inhibitory control, and spatial abilities (e.g., Krawczyk et al., 2008; Viskontas, Morrison, Holyoak, Hummel, & Knowlton, 2004; Waltz et al., 2000). Individuals with weaker cognitive capacity will have greater difficulty in reasoning by analogy and are less likely to benefit from it in educational settings (Jaeger & Wiley, 2015; Richland & McDonough, 2010).

These sources of individual variability must be taken into account when using analogy to highlight the shared structure among examples of a STEM concept. When analogical comparisons are introduced, they should be labeled as such to obviate the challenging process of noticing analogical similarity. Comparing examples highlights shared structure, but the presentation of the examples should facilitate comparison without overloading limited capacity resources. For example, shared structure can be communicated through visuospatial cues (Begolli & Richland, 2016; Matlen, Vosniadou, Jee, & Ptouchkina, 2011; Rittle-Johnson & Star, 2007). Simultaneous static presentation of exemplars frees up cognitive resources to devote to the comparison process and attend to the target material, obviating the need to hold both analogs in working memory at the same time.

In addition to keeping all analogs visible during comparison, analogical processing can be facilitated through the specific presentation style of the visual representations. Displaying visual representations such that corresponding elements are spatially aligned can support greater learning (Matlen et al., 2011; Richland et al., 2007). Color coding can also be used to emphasize the entities in different exemplars that play analogous roles (see Figure 4). Simultaneous presentation of analogs and of the layout of the entities being compared may be combined with gestures that draw attention to corresponding elements of the analogs and facilitate comparison (Richland et al., 2007; Richland & McDonough, 2010).

Visuospatial methods of emphasizing shared structure, while effective, are largely implicit. To maximize attention to crucial similarities, the correspondences between analogs should be described explicitly. This can be accomplished through verbal descriptions of the correspondences, with the instructor explicitly pointing out that two entities play the same role in analogous situations. Relational language facilitates attention to structural relational information, which contributes to learning relational concepts (Gentner, Anggoro, & Klibanoff, 2011). In addition, students can be prompted to attend to shared structure through the use of guided compare-and-contrast prompts. General prompts to compare two situations do not reliably focus attention on the most relevant dimensions of comparison, but directed compare-and-contrast instructions

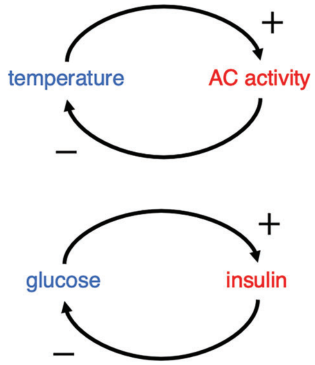


**Fig 4.** Analogous examples of feedback loops, diagrammed to visuospatially highlight shared causal structure.

(e.g., to compare instances and identify their similarities) lead students to notice shared structure (Catrambone & Holyoak, 1989; Gick & Holyoak, 1983).

As an example, a biology instructor might use this component of the analogical approach when teaching negative feedback loops, which is an inherently structural concept. Unlike cases in which a well-understood source analog is used to understand a novel target concept, the main purpose of utilizing analogy in this instance is to emphasize the shared structure that defines negative feedback loops. The two analogs selected to be aligned and compared are two examples of this relational concept. In teaching this concept using an analogical approach, it is still beneficial to begin with an example likely to be familiar to students, such as the relationship between temperature and air conditioner (AC) activity. When temperature increases, this causes the air conditioner to turn on. When AC activity increases, this causes temperature (the original quantity) to decrease. The relationship between temperature and AC activity is an example of a negative feedback loop, which occurs when an increase in a quantity (i.e., temperature) causes a later decrease in that same quantity, or when a decrease in a quantity causes a later increase in that same quantity. The top portion of Figure 4 shows a diagram of the relationship between temperature and AC activity. This diagram could be drawn on the board to free up cognitive resources in the learner.

After introducing the concept of negative feedback loops with a familiar example to capitalize on prior knowledge, instructors may then describe and diagram a second example, such as the relationship between glucose and insulin (bottom portion of Figure 4). When a person eats, glucose levels in the bloodstream rise. This causes the pancreas to secrete insulin, which lowers the level of glucose by helping the body to metabolize glucose. When diagramming this example, instructors should apply the analogical approach to highlight the shared structure that defines negative feedback loops. Specifically, instructors may draw the diagrams so that elements corresponding to one another are spatially aligned, and use color to further emphasize elements of each example that play the same role (see Figure 4). The correspondences can be explicitly pointed out. For example, an instructor might say, "Temperature corresponds to glucose because each of these things causes something else to increase," while using comparative gesture to draw students' attention to corresponding elements (Richland & McDonough, 2010). As elaborated below (principle 4), each of these modifications to instruction serves to highlight shared structure and to reduce extraneous cognitive load imposed by the comparison process.

### Explain the Connections between Semantic Information and Mathematical Operations

Bassok, Chase, and Martin (1998) and Bassok, 2001 introduced the construct of *semantic alignment*, which refers to the tendency to maintain systematic correspondences between the semantic relations that exist between real-world objects and mathematical relations between arguments of arithmetic operations. In general, students make fewer errors in constructing equations from word problems when those problems are aligned with respect to real-world relations (Martin & Bassok, 2005). For example, students readily add *a* tulips and *b* roses, but avoid adding *a* tulips and *b* vases (Bassok, 2001). Objects that are added play symmetric structural roles in the mathematical operation of addition—it does not matter if one adds tulips to roses or roses to tulips. Students therefore readily map roses and tulips onto addition (a symmetric mathematical operation) because roses and tulips come from the same taxonomic category, and taxonomically related objects play symmetric roles with respect to their joint superset (e.g., flowers). In contrast, problem solving is hindered when the mathematical structure does not fit the relations between the objects in the problem, as would be the case for a problem that requires using an asymmetrical mathematical operation, such as division, on objects that play symmetric roles (e.g., dividing roses by tulips). Semantic (mis)alignment is detected implicitly and influences event-related potentials (Guthormsen et al., 2016).

The alignment between real-world and mathematical knowledge guides the application of abstract mathematical knowledge by both students and textbook writers, not only in the United States (DeWolf, Bassok, & Holyoak, 2015; Rapp, Bassok, DeWolf, & Holyoak, 2015) but also in South Korea (Lee, DeWolf, Bassok, & Holyoak, 2016) and Russia (Tyumeneva et al., 2018). Importantly, alignments and their influence on problem solving procedures likely remain implicit in the mind of the problem solver, as they are not explicitly taught in schools. The effects of making semantic alignment more explicit have not yet been investigated. But given that highlighting shared structure demonstrably improves learning outcomes (e.g., Begolli & Richland, 2016; Richland & McDonough, 2010), it seems likely that emphasizing the alignment between a situation model and its corresponding mathematical model should facilitate understanding of the shared structure and improve problem solving.

In considering potential applications to STEM classrooms, it is useful to adopt a broad interpretation of semantic alignment. Students should be explicitly instructed about the meaning of various mathematical operations and also the mappings between mathematical and verbal representations. Not all real-world problems are aligned semantically with the formal operations that must be executed (e.g., despite the asymmetric relationship between students and teachers, the two sets must be added together to determine how many passengers will be loaded onto a bus for a field trip). Our general recommendation is to highlight the conceptual meaning of mathematical operations and then to "back-translate" from formal operations into the semantically meaningful elements of a word problem. The process of explaining the correspondences between the semantic interpretation of a problem and the relevant mathematical operations may foster understanding of the underlying conceptual structure, and enhance students' success in translating verbal models into mathematical ones.

Solid conceptual understanding of the meaning of mathematical operations contributes to mathematical reasoning in many types of problems. For example, such understanding is crucial for equation construction problems in which students must translate verbal expressions into mathematical expressions (Martin & Bassok, 2005; Simon & Hayes, 1976). In a biology class on mathematical modeling, students may be asked to translate a set of verbal assumptions that describe the state of a biological system into a series of differential equations that model how the system changes. Figure 5 summarizes four assumptions describing a simple ecosystem and the differential equations that can be constructed from them. A student in such a course may be tasked with writing a mathematical expression to represent how a population of hares, $H$, grows at a rate $b$ proportional to the current population size. Multiplication is commonly understood by novices as repeated addition, but in fact it is better conceptualized as scaling by some multiplicative factor (Devlin, 2008). The incomplete conceptualization of "multiplication as repeated addition" does not support reasoning in this situation. However, if the conceptual meaning of multiplication is addressed, it will become clear that the growth of the hare population should be represented by its current size scaled by the growth rate, and that this relationship can be expressed mathematically as multiplication, yielding $b * H$.

1. In the absence of predators, hares have a constant per-capita growth rate 0.1
2. The The rate at which a single lynx catches hares is proportional to the hare population size with proportionality constant 0.02
3. The lynx birth rate is proportional to the amount of food caught by the population with proportionality constant 0.05
4. Lynx have a constant per capita death rate 0.1

$$H' = 0.1 * H - 0.02 * L * H$$
$$L' = 0.05 * 0.02 * L * H - 0.1 * L$$

**Fig 5.** Example of model writing taught using an analogical approach. $H'$ represents how the population of hares changes and $L'$ represents how the population of lynx changes.

In addition to explaining the conceptual meaning of mathematical operations, these abstract concepts should be connected explicitly to real-world referents in the word problems from which they were generated. This can be accomplished using a combination of visual and verbal methods. As shown in Figure 5, corresponding colors can be used to visually represent the correspondence between verbal concepts and mathematical relations. Thus the first assumption (colored in blue) corresponds to the blue term in the model shown below it.

Visual cues that increase attention to correspondences between the text and mathematical structure should be accompanied by an explanation of the relevant mathematical relations in terms of their real-world interpretations. In the model shown in Figure 5, there are three different mathematical concepts at work that should be explained by an instructor. The first is addition. Mathematically, adding a positive term to a quantity increases the magnitude of that quantity. In this context, the population is the thing to which we are adding a positive term. A positive term conceptually represents something that makes a population grow in size, such as the birth of new animals. The second mathematical concept at work is subtraction. Mathematically, subtracting is the inverse of addition: subtracting a positive term from a quantity decreases the magnitude of the quantity. In this context, the semantic interpretation of subtracting a positive term is something that makes a population decrease in size, such as death of animals by predation or old age. Finally, multiplication is also relevant. Mathematically, multiplication scales a quantity by some factor. In this context, several quantities are being scaled. For example, hares, $H$, are born at a constant per-capita rate, 0.1. In other words, we expect each existing hare to increase the population of hares by a factor of 0.1. Multiplying the current population of hares by the per-capita birth rate yields a growth term that is scaled by the birth rate.

Instructors should not expect that such interpretations will come naturally to students, as many students lack conceptual understanding of even basic mathematical operations (Richland, Stigler, & Holyoak, 2012; Stigler, Givvin, & Thompson, 2010). Aligning verbal descriptions with their formal counterparts and explicating these correspondences will enhance students' conceptual understanding of mathematics, and help them connect abstract mathematical operations with real-world meaning.

**Consider Cognitive Load**

All learning imposes some cognitive load on learners. The potential benefits and harms imposed by taxing students' limited cognitive resources during learning has been debated, and a full discussion of this issue is beyond our present scope (see Kalyuga & Singh, 2016; Sweller, 2011). However, it is clear that analogical reasoning relies on the same pool of cognitive resources required for attending to to-be-learned material (e.g., Waltz et al., 2000).

Some of the load imposed by learning is *intrinsic* because of the inherent difficulty of the material being learned, and some is *extraneous* because of the particular manner in which the material is presented to learners (Sweller, 2011). Extraneous cognitive load is not inherent to the to-be-learned material, but rather varies with presentation style. It has been proposed that increasing extraneous cognitive load leaves fewer resources available to devote to the target material, and thus may harm learning for novices who have not had sufficient practice with the material (e.g., Richland & McDonough, 2010).

The instructional modifications recommended by the analogical approach involve changes to the presentation style of material, and thus risk increasing cognitive load during the learning event. We refrain from making a blanket recommendation to always seek to reduce cognitive load because some evidence indicates that delaying direct instruction and providing students with structured (but challenging) opportunities to discover principles on their own can lead to superior learning and transfer (e.g., Jacobson et al., 2017; Schwartz, Chase, Oppezzo, & Chin, 2011). More advanced students may learn better with less scaffolding (see principle 5 below; Kalyuga, 2007). It is challenging for teachers to determine how much scaffolding is optimal for a particular student. But educators need to be aware of the potential for analogy to increase the cognitive load imposed on students (whether or not that increased load is desirable).

Analogical comparison is a resource-intensive process, but extraneous load may be reduced when appropriate. As noted in the section on highlighting structure, the extraneous cognitive load incurred by analogical comparison of exemplars can be minimized through small changes to the instructional delivery. First, the exemplars that are the focus of the comparison should be represented visually and presented simultaneously whenever possible. Corresponding elements of the exemplars should be aligned spatially, and may be written in corresponding colors to further highlight that they play analogous roles in their respective situations (see Figures 4 and 5). Gesturing between elements that play corresponding roles in different analogs also facilitates comparison (Richland & McDonough, 2010). In addition to these visuospatial methods, cognitive load may be reduced further by explicitly pointing out correspondences during initial learning (Novick & Holyoak, 1991). For example, if a biology instructor is teaching negative feedback loops using two analogous examples of the concept, they can describe the correspondence explicitly and justify it (e.g., in Figure 4, temperature corresponds to glucose because each of these things causes something else to increase).

**Encourage Generation of Inferences**

Studies of human memory have shown that generating information usually leads to better retention than passive study (for a review see Bertsch, Pesta, Wiscott, & McDaniel, 2007). This *generation effect* has also been demonstrated in educational settings (Metcalfe & Kornell, 2007). Recent work suggests that generating analogical inferences (rather than simply verifying them) may be similarly beneficial. Vendetti, Wu, and Holyoak (2014) compared the effects of generating solutions to semantically distant four-term verbal analogies to passively viewing and evaluating completed analogies. Generating solutions to distant analogies selectively fostered the induction of a relational mindset that fostered attention to relational information in a subsequent analogy task using completely different relations (see also Andrews & Vann, 2019; Simms & Richland, 2019). These findings suggest that generating relational information may encourage participants to attend to it.

When students are first introduced to a concept using analogous examples, tasking them with generating the mappings between corresponding elements of the examples is likely to impose too great a cognitive load. However, this technique may be introduced later on in a lesson, when students have some familiarity with the concept. Low-knowledge learners typically need significant scaffolding (see principles 2 and 4), but these techniques may lose their efficacy for high-knowledge learners. This transition in effective instructional techniques from low- to high-knowledge learners has been termed the "expertise reversal" effect (Kalyuga, 2007), and the analogical approach fits within this framework. Early on, instructors should provide explicit guidance on analogical comparison to prevent the comparison from overwhelming limited cognitive resources. As learners gain expertise, however, their need for instructor guidance is likely to be reduced. Generating mappings among analogs and drawing appropriate inferences directs attention to relational structure, which gives proficient students the opportunity to practice attending to the important relational information without direction from an instructor. This type of activity resembles testing and real-world contexts in which instructor guidance is conspicuously absent. Further, previous research suggests that generating the underlying structure will lead to greater retention of that structure (Bertsch et al., 2007).

This component of an analogical approach to instruction can be illustrated using the example of negative feedback loops. As discussed previously, negative feedback loops occur when an increase in a quantity causes a later decrease in that same quantity, or when a decrease in a quantity causes a later increase in that same quantity. After the concept has been introduced and the instructor has explained some examples of it (see Figure 4), students will have some familiarity with the concept. At this point, the instructor may describe another example of a negative feedback loop and instruct students to align it with the initial examples. The relationship between a population of tuna and a population of sharks is another case of a negative feedback loop: when the tuna population increases, it causes the shark population to increase because there is more food available. When the shark population increases, this causes the tuna population to decrease because they will be eaten by more sharks. If students properly align the tuna with glucose and temperature, they will be generating the shared structure that defines negative feedback loops.

## CONCLUSIONS

Analogy is a powerful and flexible educational tool. Although we have focused on applications in STEM fields, the benefits conferred by analogy are not limited to these domains, or to the relatively advanced types of concepts we have discussed as examples. Young children are drawn to superficial similarities (Richland, Morrison, & Holyoak, 2006), but with appropriate guidance from an educator highlighting relational information, children may also benefit from analogical instruction (Holyoak et al., 1984). Further, education in non-STEM fields such as political science (Spellman & Holyoak, 1992), business (Gentner et al., 2003), and law (Lamond, 2016) may also be improved by inclusion of analogies. Much of the subject matter in such fields may be better suited to verbal than visual representation, but the same key principles can be applied. Taken together, the principles suggested here provide a framework to guide the application of analogy in classroom settings so as to foster greater conceptual understanding and transfer.

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