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Semantic alignment across whole-number arithmetic and rational numbers: evidence from a Russian perspective

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ABSTRACT

Solutions to word problems are moderated by the semantic alignment of real-world relations with mathematical operations. Categorical relations between entities (tulips, roses) are aligned with addition, whereas certain functional relations between entities (tulips, vases) are aligned with division. Similarly, discreteness vs. continuity of quantities (marbles, water) is aligned with different formats for rational numbers (fractions and decimals, respectively). These alignments have been found both in textbooks and in the performance of college students in the USA and in South Korea. The current study examined evidence for alignments in Russia. Textbook analyses revealed semantic alignments for arithmetic word problems, but not for rational numbers. Nonetheless, Russian college students showed semantic alignments both for arithmetic operations and for rational numbers. Since Russian students exhibit semantic alignments for rational numbers in the absence of exposure to examples in school, such alignments likely reflect intuitive understanding of mathematical representations of real-world situations.

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KEYWORDS Semantic alignment; mathematical reasoning; cross-national comparison

Introduction

When applying mathematics to real-world situations, students must understand how to construct mathematical representations of the situations they encounter. Educators try to teach this process by approximating real-world situations with word problems. For example, a division word problem may require dividing a set of flowers among a set of vases, where the mathematical concept of division requires allocation of equal numbers of flowers to

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each vase. These types of problems require students to generate "situation models" that are analogous to the appropriate mathematical representations, or "mathematical models" (Kintsch & Greeno, 1985).

Semantic alignment in understanding mathematical problems

Previous research has shown that the process by which people coordinate situation and mathematical models is often guided by *semantic alignment* (e.g., Bassok, Chase, & Martin, 1998; Bassok, Wu, & Olseth, 1995; Dixon, Deets, & Bangert, 2001). Bassok et al. (1998) interpreted semantic alignment as a heuristic for relating situation models to mathematical models. As a heuristic, its use may be helpful in many circumstances, but in others may cause interference. Semantic alignment is a process of analogical mapping between semantic relations (often either categorical or functional) implied by the objects in the problem situation, and potential mathematical relations (Bassok et al., 1998; for a review see Holyoak, 2012). In the flower and vases example (see Figure 1), flowers and vases evoke the functional relation *contain* [*flowers, vases*], which is asymmetric (because vases normally *contain* flowers and not vice versa). This functional relation aligns structurally with the mathematically asymmetric relation *divide* [*dividend, divisor*]. The semantic alignment for the

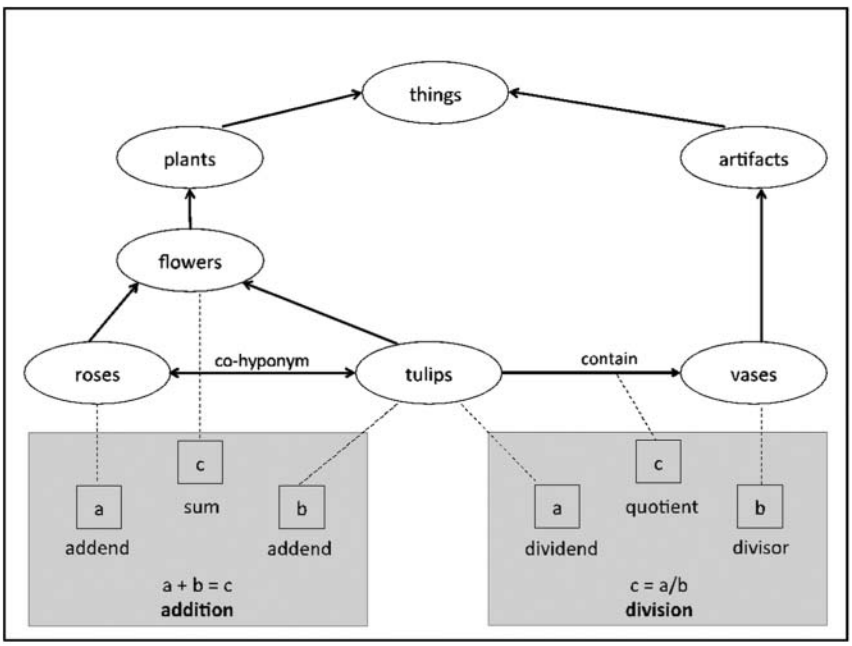

Figure 1. Semantic alignments between addition and division arithmetic relations and conceptual structures of items in the world as described by Bassok et al. (1998). Reprinted with permission from Guthormsen et al. (2016).

objects tulips and roses naturally evoke their shared categorical superset relation, *both-flowers* [*tulips*, *roses*]. Unlike the *tulips-vases* pair, tulips and roses play symmetric roles in the 'both-flowers' relation, which aligns structurally with the symmetric mathematical relation *add* [*addend1*, *addend 2*].

Bassok et al. (1998) found that this type of semantic alignment is highly systematic among American college students. These researchers asked college students to construct addition or division word problems given various pairs of object sets. Students preferred to generate addition problems for categorically related (i.e., symmetric) sets (e.g., "6 *tulips* plus 2 *roses* gives you how many flowers?"), but preferred to generate division problems for asymmetrical sets (e.g., "How many tulips per vase do you have with 21 *tulips* and 3 *vases*?"). This pattern of alignment is prevalent across middle school, high school and college students asked to solve arithmetic word problems (Fisher, Borchert, & Bassok, 2011; Martin & Bassok, 2005). Other evidence suggests that the impact of such alignments is relatively automatic, as an arithmetic relation is processed more quickly if it is preceded by an object pair exhibiting an alignable relation (Bassok, Pedigo, & Oskarsson, 2008). In addition, distinct patterns of event-related potentials are evoked when semantic and arithmetic relations are misaligned (Guthormsen et al., 2016).

A different type of semantic alignment, based on the distinction between discrete and continuous quantities, has also been observed. Bassok and Olseth (1995) found that the discreteness versus continuity of the entities described in a problem (e.g., salary increases versus increases in the value of a coin in $/year) affects the way people represent problem structures, and therefore impacts transfer of learned solutions. The same alignment process affects how people choose a format for rational numbers (fractions versus decimals) to represent discrete versus continuous entities (Rapp, Bassok, DeWolf, & Holyoak, 2015). Figure 2 shows the hypothesised alignment between rational number type and entity type. Rapp et al. (2015) found that both college students and textbook writers show a preference for representing relations between discrete or countable entities with fractions (e.g., 3/4 of the marbles), and for representing magnitudes or measures of continuous entities with decimals (e.g., .75 L of water). Thus, alignment not only influences the generation of concrete instantiations of mathematical representations (the focus of the present paper), but also the generation of mathematical representations to match given concrete situations.

Cross-national comparisons of semantic alignment

The nature of semantic alignment highlights the more general issue of how instantiation of an abstract concept is affected by concrete details of a situation. Does an abstraction necessarily sever connections to concrete instantiations, or does it remain linked to them? If the latter, what factors determine

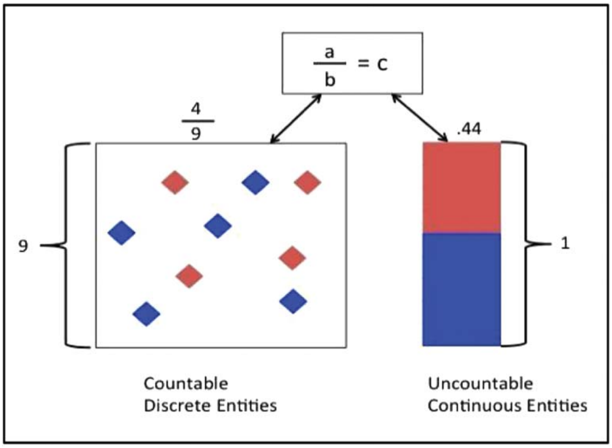

Figure 2. The bipartite structure of a fraction maps to discrete subsets in a visual display (left), whereas the decimal equivalent represents a one-dimensional magnitude (right). Reprinted with permission from Lee et al. (2015).

the linkage? A critical theoretical question concerns whether people's preferred semantic alignments reflect a basic understanding of mathematical representations as analogical models of real-world situations, or reflect a history of specific learning experiences and therefore correlate with instructional practices, language or culture. As an initial effort to address this question, Lee, DeWolf, Bassok, and Holyoak (2015) investigated whether the alignment between rational numbers and entity type, first shown for students in the United States, would also hold for students in South Korea, a nation that differs greatly from the United States in language and in educational and cultural practices. Lee et al. found the same pattern of semantic alignments (for both Korean textbooks and college students) as Rapp et al. (2015) had found for American students. Of course, a single cross-national comparison cannot definitively rule out the possibility that some aspect that is shared across the two nations is critical to the emergence of semantic alignment. In addition, given that semantic alignments in both nations appeared in textbooks, it remains unclear whether semantic alignments observed in the performance of college students are due to exposure to such examples, or whether both students and textbook writers are guided by the same basic understanding and use of mathematical representations. In other words, the observed correlation has not been established as a causal connection.

Distinguishing characteristics of the Russian math curriculum

The goal of the present paper is to address this question by expanding the cross-national exploration of semantic alignment to the Russian Federation, where the math curriculum differs radically from that found in either the USA or South Korea. Two major differences are noteworthy. First, the Russian curriculum strongly emphasises abstraction, a focus that may decrease the importance of alignment with concrete semantic relations. Second, the Russian approach emphasises measurement, a practice that may undermine the distinction between continuous and discrete quantities.

Abstraction

The math curriculum in the Russian Federation is built on theoretical foundations that specify a strict sequence of topics and their interconnections. Students learn math structures such as function, group, magnitude, and number, which run throughout the entire math curriculum (Bourbaki, 1950; Kolmogorov, 1990). It is assumed that students should learn each concept iteratively, passing through successive levels of generalisation, from very concrete manipulation to completely abstract reasoning. Similar ideas about the value of a rapid transition from manipulating concrete objects to abstract math have arisen out of the Piagetian approach (Bruner, 1990; Piaget, 1970).

Importantly, the dominance of a unified theoretical approach has fostered homogeneity in teaching methods across Russia. Only a few math textbooks are approved for school usage by the Ministry of Education. Each textbook is accompanied by a teacher's book that provides instructions and supplemental materials to teach every topic. A pedagogical method based on the idea of math structures, termed the "developmental education" approach (Corry, 1992; Davydov, 1982), is especially prominent in the Russian Federation. This approach (for a full discussion see Davydov, 1990; Kelly & Washtell, 1996; Lamon, 1993; also Sophian, 2004) encourages learning that all basic math operations are conceptually interrelated, applying measurement units to discrete objects in exactly the same way as to continuous quantities.

Additional features of the Russian math curriculum differ sharply from those of the US and Korean curricula, and may influence the degree to which students' mathematical thinking is guided by semantic alignment. First, Russian children are taught the beginnings of algebra as early as elementary school, including skills such as building and solving equations. Although they are given many word problems involving real objects, children are encouraged to abstract their solutions into mathematical expressions. Although abstract thinking in math is also promoted in the US school system, this emphasis is nowhere near as strong as in Russia, where a focus on abstraction is promoted at all times (notably, in official syllabi, in textbooks and in learning goals for students). At the state level, teaching abstraction is directly specified as an element of the compulsory mathematical minimum content for primary school (Mullis, Minnich, Arora, Centurino, & Castle, 2012, p. 761; see also Ministry of Education and Science of the Russian Federation, 2004). At the textbook level, generalisation and abstraction are supported in the most popular textbooks for elementary school (e.g., Moro, Bantova, Beltyukova, Volkova, & Stepanova, 2011, 2016). At the level of everyday practice, math teachers typically ask children to represent the abstract structure of a problem by drawing a "schema" for the problem, using a letter to represent "any number" or "unknown". This focus on abstraction in the Russian curriculum might be expected to diminish the impact of semantic alignment, which depends on the more concrete properties of the objects involved in mathematical problems.

Measurement

Children in Russia learn that natural numbers are not only countable, but also constitute ordered magnitudes. This approach applies to all kinds of real numbers, thus maintaining consistency throughout the entire curriculum. The strong focus on magnitude and measurement in the Russian curriculum can be also observed systematically at all levels of math education. Compulsory minimum content for primary school (Grades 1–4) mathematics education in the Russian Federation explicitly includes topics such as, "order subjects based on different attributes, such as length, weight, capacity, and time; learn units of length, weight, capacity, and time; and learn relations between units comparing and ordering homogeneous quantities" (Mullis et al., 2012, p. 761; see also Ministry of Education and Science of the Russian Federation, 2011). At the level of instructional strategies, students are taught that units of measurement are interchangeable and can be applied to discrete objects in exactly the same way as to continuous quantities. At the level of everyday practice, students are encouraged to measure and compare different characteristics of physical objects, acquiring a concept of real number as a consequence of these activities. Children learn that units of measurement are arbitrary and therefore can be replaced by one another if their relationships are held constant. For examples, the unit of measurement can be either 1 metre or 1 centimetre as long as we keep in mind their relation (1 m = 100 cm). Similarly, discrete objects can be grouped based on their quantity. To compare groups requires using units of measurement; as in the case of continuous magnitudes, such units are arbitrary and interchangeable. For example, to measure and compare groups of apples, we might use a single apple (a small unit of measure) or a standard set of ten apples (a larger unit of measure), keeping in mind their relation (1 set-of-10-apples = 10 single apples). This strong focus on magnitude and measurement in the Russian curriculum would seem likely to promote general dominance of continuous over discrete magnitudes, thereby diminishing semantic alignment of discrete magnitudes with fractions and continuous magnitudes with decimals.

In addition to the focus on measurement, fractions and decimals are introduced simultaneously within the Russian curriculum (in contrast to the USA, where fractions are typically introduced to students at least a year prior to decimals). In popular Russian textbooks, fractions and decimals are introduced one after another in the same grade (e.g., Vilenkin, Zhokhov, & Schwartzburd, 2013, p. 133, p. 180; Dorofeev & Peterson, 2011, part 2, p. 4, p. 146), separated by at most one topic (e.g., Zubareva & Mordkovich, 2013, p. 89, p. 179). Coupled with the strong focus on measurement, which unifies continuous and discrete entities, simultaneous learning of the two notations for rational numbers might further reduce any selective semantic alignment of number format with entity type.

Goals of the present study

We examined whether the patterns of semantic alignments for basic arithmetic operations and for rational numbers, previously found in the textbooks and in the performance of students in the USA (in the latter case, in South Korea as well), will also be observed in Russia. We examined whether the focus on abstraction would diminish the magnitude of semantic alignment in textbook word problems and in subsequent students' performance. We also examined whether the Russian textbooks use continuous quantities in word problems for both fractions and decimals. If this is the case, it is highly unlikely that Russian students learn in school to align fractions with discrete quantities and decimals with continuous entities. To the extent that semantic alignments are based on formal education, Russian college students may have no preference for using fractions or decimals to represent continuous quantities.

It is possible, however, that some other mechanism besides formal textbook instruction elicits semantic alignment in adults. For example, quantitative problems encountered in everyday life may exhibit systematic pairings between quantity types and formats for rational numbers. It is also conceivable that semantic alignment could emerge because fractions are interpreted as discrete quantities whereas decimals are interpreted as continuous (Rapp et al., 2015). If semantic alignment can arise via multiple mechanisms, Russian students may behave like American and Korean students – they may prefer to use fractions rather than decimals to represent discrete quantities. That is, it is possible that Russian students will show a pattern of semantic alignments for rational numbers (favouring fractions more for discrete than continuous entities) even though such alignments were not learned in school. If so, this would provide evidence that semantic alignment arises from basic psychological processes, even in the absence of correlations encountered in the course of formal instruction.

The organisation of the paper is as follows. We first examined whether the pattern of semantic alignment for basic arithmetic operations holds for Russian math textbooks (arithmetic textbook analysis) and for undergraduate students (Experiment 1). We then assessed if a pattern of alignment is found for rational numbers, both for textbooks (rational number textbook analysis) and for undergraduate students (Experiment 2).

Arithmetic textbook analysis

In order to determine whether Russian textbooks show semantic alignment for basic arithmetic problems (addition: symmetrical, division: asymmetrical), we conducted a textbook analysis of 4th and 5th grade textbooks. Bassok et al. (1998) found that an overwhelming majority (96%) of the problems in an American textbook series from 1st through 8th grades showed clear semantic alignment. The present analysis examined whether the same pattern holds for popular Russian textbooks.

Method

Materials

We analysed two textbooks for Grades 4 and 5 (Moro et al., 2011; Vilenkin, Zhokhov & Schwartzburd, 2008), which have a large market share (more than 50%) (Demidova et al., 2013) and are widely used across Russia (Roslova, 2009).[1] Our analysis was restricted to textbooks for Grades 4 and 5 because in Russia word problems based on real objects are most often included in math textbooks for these grades. Word problems (i.e., problems based on real objects) also occur in textbooks for Grades 6 and 7, but the number of such problems declines drastically after Grade 5. We analysed all word problems involving addition/subtraction or else division/multiplication, a total of 740 problems (addition/subtraction = 419, division/multiplication = 321). For this analysis, we considered every word problem, both those with whole numbers and with fractions.

Problem coding

The problems were coded in a manner similar to the system developed by Bassok et al. (1998). First, problems were parsed into subproblems, each involving only a single arithmetic operation, and divided into addition or subtraction problems (combined) versus division and multiplication problems (combined). Then, the semantic relations between elements of the problems were classified as either symmetric (e.g., red and blue marbles) or asymmetric

[1]According to a TIMMS report (Demidova et al., 2013), the textbook by Moro et al. (2011) has over 50% market share. According to data provided by the Russian State Library (www.rsl.ru), the text by Vilenkin et al. (2008, 2013) has been published since the 1990s; 75,000 copies were printed in 2008 (24th edition) and 50000 copies were printed in 2015 (34th edition). The latter textbook has been translated into other languages spoken in the Russian Federation (e.g., Tatar). Other textbooks used in our analysis are also in widespread use. For example, the text by Makarychev et al. (2008), analysed in the next section, has been published every year for more than 15 years, with about 20,000 copies each printing.

Table 1. Examples of problems identified in analysis of Russian textbooks on arithmetic operations and semantic relations.

Arithmetic operation	Semantic relations	Example of textbook problem
Addition/ subtraction	Symmetry	Two workers work at the same workpieces. The first worker serves 5 machine tool stations, preparing 7 workpieces per hour. The second worker serves 4 machine tool stations, preparing 15 workpieces per hour. How many workpieces will they produced together in 8 hours?
	Asymmetry	Two cisterns contain 119.8 tons of gasoline. The first cistern contains 1.7 times more gasoline than the other one. How many tons does each cistern contain?
Division /multiplication	Symmetry	There are 840 packs of black tea in a warehouse, and the amount of packs of green tea is 3 times less. How many packs of green tea are in the warehouse?
	Asymmetry	10 packs of books were delivered to a school. Each pack contains 20 books. How many books were delivered to the school?

(e.g., flowers and vases). One Russian researcher coded all problems with respect to these four categories. A second coder (a bilingual Russian-English speaker living in the United States), blind to the original coder's judgments, coded a subset of the problems to assess interrater reliability. The second coder analysed a random subset of 20% of the original problems. The two coders agreed on 96% of the problems, indicating that the first round of coding was highly reliable. A third coder broke the tie for those problems for which there was a disagreement. Table 1 shows examples of the coded problems in the textbooks.

Results and discussion

Table 2 shows the relative percentage of symmetric and asymmetric object sets that were related by either addition or division. Almost all of the addition and subtraction problems (99%) involved symmetric object pairs, whereas the great majority of the division and multiplication problems (88%) were asymmetric. There was a significant relation between arithmetic operation and semantic structure (χ^2 (1) = 578.797, $p < 0.001$). This finding replicates the pattern of results observed in American textbooks, in which 97% of addition problems involved symmetric relations and 94% of division problems involved asymmetric relations (Bassok et al., 1998, Experiment 3).

Table 2. Alignments between arithmetic operations and semantic relations in Russian math textbooks.

	Addition/subtraction	Division/multiplication
Symmetric	415 (99%)	39 (12%)
Asymmetric	4 (<1%)	282 (88%)
N	419 (100%)	321 (100%)

Experiment 1

The textbook analysis revealed that Russian textbook writers follow the same pattern of semantic alignment as do American educators in selecting real-world entities to construct arithmetic word problems. The goal of Experiment 1 was to assess whether undergraduate students in Russia, like those in the USA, also honour this pattern of alignment. This experiment was modelled after Experiment 2 reported by Bassok et al. (1998). In the original study, students were given pairs of object sets that were related either symmetrically (e.g., *tulips– daffodils*) or asymmetrically (e.g., *tulips–vases*), and were asked to generate either addition or division word problems involving the given objects. As in the original study, students in Russia were also given set–subset object pairs (e.g., *tulips–flowers*). In the original study, when the object relations were semantically aligned with the required operation, students typically generated "mathematically direct" problems (e.g., apples + oranges; tulips/vases). Interestingly, when the object relations were misaligned with the required operation (e.g., apples and oranges for division; tulips and vases for addition), students often generated more complex "semantic escape" problems in which object relations were aligned with the mathematical relations. For example, a "semantic escape" addition problem for the asymmetric pair tulips and vases might be: (tulips + daisies)/vases. The present Experiment 1 examined whether Russian college students also generate such semantic-escape problems.

Method

Participants

Participants were 77 undergraduate students from the Faculty of Psychology, National Research University Higher School of Economics (72 females and 5 males). The experiment was conducted at the beginning of the school year and only 1st-year students were selected, thus minimising the influence of university education on students' performance. All participants were tested together in one group.

Materials

The materials were adapted from Bassok et al. (1998) Experiment 2. Some object pairs were changed to accommodate a translation to Russian. Three types of object pairs were used:

(1) *Symmetric*: object pairs from the same taxonomic category that belonged to a common superset (e.g., dolls – balls).
(2) *Subset–set*: object sets that had a set–subset structure (e.g., boys–children).
(3) *Asymmetric*: object pairs that play asymmetric structural roles in a functional relation (e.g., boys – schools).

Table 3. Object pairs used in Experiment 1.

Semantic relationship		
Symmetric	Subset–set	Asymmetric
Boys–girls	Boys–children	Boys–schools
Oculists–dentists	Oculists–doctors	Oculists–hospitals
Guitarists–drummers	Guitarists–musicians	Guitarists–bands
Apples–pears	Apples–fruits	Apples–crates
Balls–dolls	Balls–toys	Balls–boxes
Tulips–daffodils	Tulips–flowers	Tulips–vases

Table 3 shows the pairs used for the symmetric, subset–set and asymmetric conditions. Half of the pairs of each type involved people and the other half involved inanimate objects. Bassok et al. (1998) noted that subset–set pairs might plausibly be aligned with the addition schema, but only (see below for description of coding scheme) if the complementary subtraction operation is used (i.e., subtracting the subset from the overall set, as in number of children minus number of boys). Subset–set pairs might also be aligned with division, assuming the subset is mapped to the dividend and the set to the divisor to form a proportion (e.g., number of boys divided by number of children). Bassok et al. found that the performance pattern on construction tasks for subset–set pairs was intermediate between that observed for symmetric and asymmetric pairs.

The 18 pairs were randomly divided into three equivalent construction booklets, each of which consisted of six different pairs: one symmetric pair of people, one symmetric pair of objects, one asymmetric pair of people and locations, one asymmetric pair of objects, one subset–set pair of people and one subset–set pair of objects. The order of the symmetric, asymmetric and subset–set pairs in each booklet and the order of the words in each pair varied between booklets. The first page of each booklet contained the name of the operation (addition or division) that the participant was to apply to all pairs of objects in the booklet when creating math word problems.

Procedure

Participants were randomly assigned to receive an addition ($N = 40$) or a division ($N = 37$) booklet. They were asked to create math word problems involving addition or else division for each of the six object pairs in the booklet. The allotted time was limited to 20 minutes.

Results and discussion

Problem coding

In total, participants constructed 240 addition word problems and 222 division word problems. All constructed problems were assigned to one of four categories, as described below. The coding system used was adapted from that outlined by Bassok et al. (1998). Based on the equations required for the problems' solution, the generated problems were coded into the following four categories:

(1) Mathematically direct (MD): problems in which the equation related the given sets directly by the required arithmetic operation ($a + b = c$ for addition, $a/b = c$ for division) and did not involve any other mathematical operation (e.g., for *tulips-daffodils*, Addition condition: "*A bouquet consists of 13 tulips and 4 daffodils. How many flowers are in the bouquet?*"; for *apples-boxes*, Division condition: "*There are 40 apples in 5 boxes. How many apples are in each box?*").
(2) Complex mathematically direct (Complex MD): problems in which the sets were related directly by the requested operation but included further computation. This category includes problems where students introduced variation into their problems that was unrelated to semantic alignments (e.g., for *boys-girls*, Addition condition: "*There are 15 boys and 13 girls in one group, and 17 boys and 10 girls in another group. How many children are in both groups?*"; for *tulips-daffodils*, Division condition: "*There are 10 vases on the wedding table. Half of vases hold 30 tulips, and another half of vases hold 45 daffodils. How many tulips and daffodils are in each vase?*").
(3) Semantic escape (SE): problems where some requirements were not fulfilled. For example, students sometimes used the semantically alignable operation instead of the requested but semantically nonalignable operation (e.g., for *tulips-vases*, Addition condition: "*There are 12 tulips in four vases. How many tulips are in one vase?*"). As another type of example, the constructed problem might incorporate only one of the two specified objects, or only one object instead of the two required (e.g., for *apples-pears*, Division condition: "*Apples and pears are to be shared among four people equally. If there are eight apples, how many of them would each person receive?*").
(4) Other: problems that constituted irrelevant answers, such as using a non-arithmetic operation, or making jokes (e.g., "*There are four bands with three guitarists and one drummer in each. How many musicians does it take to change a light bulb?*").

Note that although students were asked to construct word problems using addition or else division, many of them constructed word problems involving the mathematically complementary operations. Following the system of problem coding used by Bassok et. al. (1998), we coded problems involving the mathematically complementary operations of addition and subtraction as "addition", and coded the mathematically complementary operations of division and multiplication as "division".

Figure 3 shows percentages of MD, complex MD, SE and other problems constructed for each pair type by participants. The top panel presents data

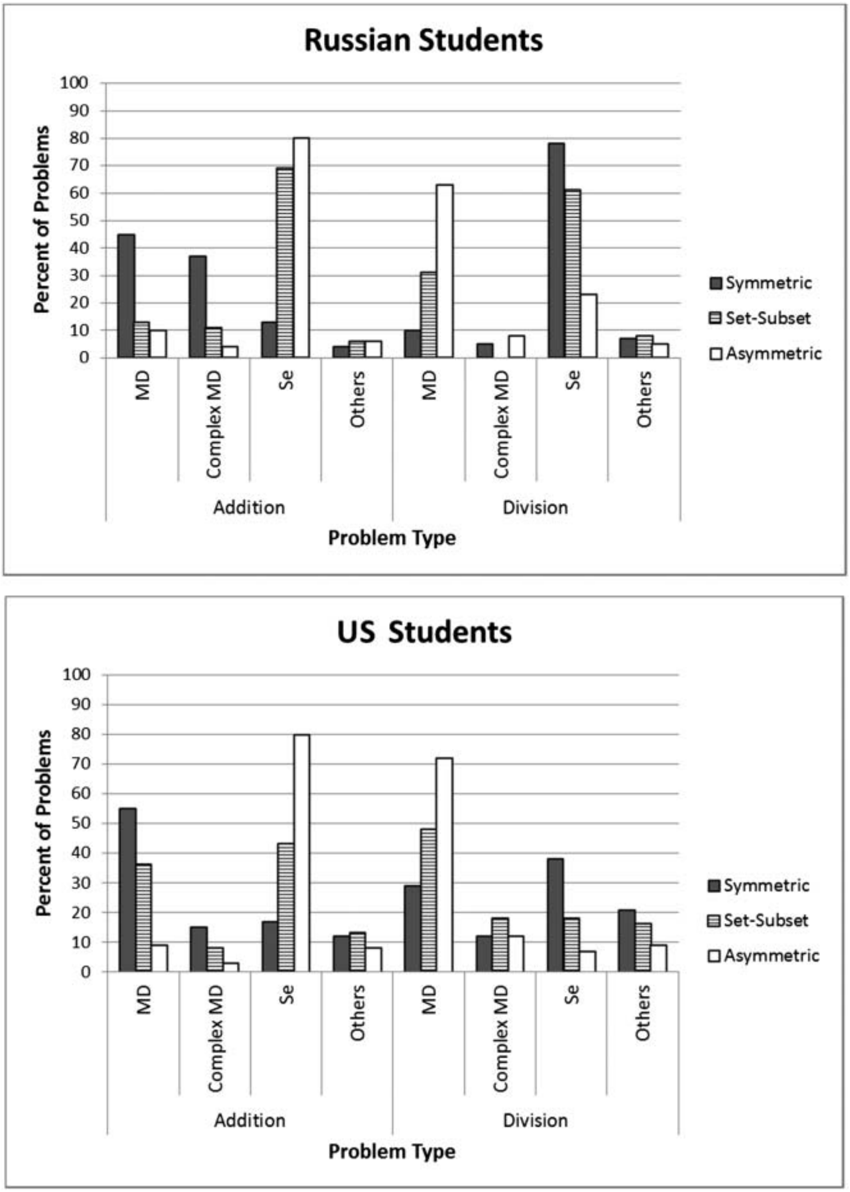

Figure 3. Percentages of MD, complex MD, SE, and other problems constructed in Addition and Division conditions for symmetric, set–subset and asymmetric relations. Top: current results for Russian students; bottom: results for US sample (data from Bassok et al., 1998, p. 117).

from the present Russian sample, and the bottom panel shows data from the comparable American sample tested by Bassok et al. (1998).

In the Addition condition, the relative frequency of MD problems was higher for aligned symmetric pairs (45%) than for misaligned asymmetric pairs (10%), with the frequency for subset–set pairs falling in between (13%). The pattern for generating complex MD problems generally matched that for MD problems, being higher for symmetric pairs (37%) than asymmetric pairs (4%), with subset–set pairs intermediate (11%). The opposite pattern was observed for SE problems, where the percentage was higher for asymmetric pairs (80%) than for symmetric pairs (13%), with subset–set pairs intermediate (69%).

These patterns generally reversed in the Division condition. The percentage of MD problems was far higher for aligned asymmetric pairs (63%) than for misaligned symmetric pairs (10%), with the frequency for subset–set pairs falling in between (31%). Complex MD problems were seldom generated in the Division condition (8%, 0% and 5% for symmetric, subset-set, and asymmetric pairs, respectively). The relative frequency of SE problems in the Division condition was higher for misaligned symmetric pairs (78%) than aligned asymmetric pairs (23%), with subset–set pairs intermediate (61%).

To test the hypothesis that only a small number of participants were responsible for the construction of non-MD problems (i.e., semantic escape and others problems), a non-parametric Mann–Whitney test was conducted. This test focused on the symmetric and asymmetric conditions, since these represent the clear extremes. The relation between object symmetry and problem structure at the level of participants was tested. In the Addition condition, the mean difference between the number of MD and non-MD problems that each participant constructed was 1.3 (SE = 0.18) for symmetric pairs and −1.38 (SE = 0.17) for asymmetric pairs (Mann–Whitney $U = 148$, $p < .01$). In the Division condition, the comparable mean difference was −1.35 (SE = 0.21) for symmetric pairs and 0.22 (SE = 0.25) for asymmetric pairs (Mann–Whitney $U = 328.5$, $p < .01$). Thus, in both conditions, participants were more likely to construct MD problems when the required operation was alignable with the semantic relation between sets.

In general, the results obtained from the Russian sample closely matched those obtained using the US sample tested by Bassok et al. (1998). In particular, the percentage of MD problems was higher for symmetric pairs when the required operation was addition and for the asymmetric pairs when the required operation was division, whereas the percentage of semantic escape problems showed the opposite pattern. In both data-sets, set–subset pairs generally produced a pattern intermediate between that of the symmetric and asymmetric conditions.

Rational number textbook analysis

The arithmetic textbook analysis and Experiment 1 provides a close Russian replication of the arithmetic semantic alignment between addition and division and type of semantic relation (symmetric versus asymmetric) previously shown in the USA. An additional textbook analysis and experiment were performed to determine whether or not the semantic alignment between formats for rational numbers and entity types (fractions: discrete; decimals: continuous) holds across textbooks and students in Russia. As discussed earlier, there was reason to hypothesise that this pattern of alignment for rational numbers would *not* hold for textbooks, due to the near-exclusive focus on continuous entities in Russian education. We therefore conducted an analysis of the three most popular Russian textbooks, using the same approach as was employed by Rapp et al. (2015) in their analysis of American textbooks. We limited this analysis to Grades 5–7 only, since in Russia both fractions and decimals are introduced in the 5th grade, and textbooks after 7th grade include too few word problems to permit meaningful analyses.

Method

Materials

We examined three textbooks for Grades 5, 6 and 7 (Vilenkin, Zhokhov, & Schwartzburd, 2005, 2008; Makarychev, Mindjuk, Meshkov, & Feoktistov, 2008). Books by these authors (for different grades of secondary and higher school) are recommended by the Russian Ministry of Education. They have a very large circulation, and are chosen by great number of Russian teachers. According to a TIMSS report (Demidova et al., 2013), about 50% of math teachers adopt the textbook by Makarychev et al. (2008) (also see Footnote 1).

Only word problems containing fraction or else decimal numbers were analysed. Problems consisting of several parts or containing several fraction/entity pairs were coded separately as different problems. In total, 476 problems were examined, of which 216 included decimals and 260 included fractions. This is greater than the number of problems found for 5–7th grades in the USA (fractions: 180, decimals: 239) and Korea (fractions: 159, decimals: 115). However, fraction problems are introduced much earlier in the US curriculum (as early as kindergarten), and thus a greater number of problems are given over that earlier period.

Problem coding

In order to code problems into categories, we used the scheme outlined by Rapp et al. (2015; also DeWolf, Bassok, & Holyoak, 2015). First, word problems were divided into decimal or fraction problems (problems that contained both were excluded). Then, word problems were classified as continuous or discrete, depending on the type of entity used in the problem. All units in problems were metric. A problem was coded as continuous if it included an object linguistically referred to in English as a mass noun (e.g., length, weight or speed), or an object that cannot be broken down into natural equal units

Table 4. Examples of problems using different entity types identified in Russian textbook analysis.

Entity type	Unit types	Example of textbook problem
Continuous	Mass nouns, units of measure (e.g., kg, liter, degrees Celsius, etc.)	There was 54.5 kg of cereal in the first sack, 1.7 times less kilos in the second sack than in the first one and 2.6 times more kilos in the third sack than in the second one. How many kilos of cereals there were in the three sacks altogether?
		A road which was 820 meters long was repaired in three days. The workers finished 2/5 of this road on Tuesday and 2/3 of the remaining part they did on Wednesday. How many meters did the workers repair on Thursday?
Discrete	Collective nouns (class, group, etc.), slices of a mass (pieces of a cake), discrete objects (e.g., a balloon, a stone, etc.)	22 dogs were taken to an Arctic base. 5/11 of those dogs were harnessed for a trip. How many dogs were left?
		There are 140 pages in a book. Alyosha has read 0.8 of them. How many pages has Alyosha read?

(e.g., a field or a road). A problem was coded as discrete if it included objects that cannot be naturally divided into similar parts (e.g., a balloon, a stone, etc.). Continuous objects that were broken down into equal-sized pieces that could be counted (e.g., equal pieces of a cake) were considered "discretised" and also included in the discrete category. This decision was based on findings from our previous work (e.g., DeWolf et al., 2015), in which people responded similarly to discrete and discretised stimuli.

Examples of problems and their coding are shown in Table 4. One researcher coded all problems based on the coding scheme outlined. A second coder (the same Russian-English speaker who assisted with the textbook analysis for arithmetic), blind to the original coder's judgments, coded a subset of the problems to assess interrater reliability. The second coder analysed a random subset of 20% of the original problems. The two coders agreed on 94% of the textbook problems, indicating high reliability. A third coder broke the tie for those problems for which there was a disagreement.

Results and discussion

The results of the textbook analysis are shown in Table 5. The great majority (91%) of the decimal problems used continuous entities. However, in stark

Table 5. Frequencies of alignments between formats for rational numbers and entity types in Russian math textbooks.

	Decimal	Fraction
Continuous	197 (91%)	225 (87%)
Discreet	19 (9%)	35 (13%)
N	216 (100%)	260 (100%)

contrast to findings in comparable analyses of similarly popular textbooks used in the USA and South Korea (described by Rapp et al., 2015, and Lee et al., 2015, respectively), most fraction problems (87%) also used continuous entities. A test of independence between number and object type showed that the two factors were not reliably associated, $\chi^2(1) = 2.55$, $p = 0.11$. We also calculated confidence intervals to assess the frequency difference for decimals versus fractions in word problems containing continuous or discrete entities, using the Wilson score interval method (Brown, Cai, & Dagupta, 2001). The 95% confidence intervals for continuous/decimal ($n = 197$) and continuous/fraction ($n = 225$) problems were [0.86–0.94] and [0.81–0.90], respectively. Confidence intervals for discrete/decimal ($n = 19$) and discrete/fraction ($n = 35$) problems were [0.05–0.13] and [0.09–0.18], respectively. Thus, based on 95% confidence intervals, the hypothesis that continuous entities were used in both decimal and fraction problems with equal frequencies cannot be rejected. The same conclusion holds for discrete entities. Thus, the findings from the textbook analysis indicate that students in Russia are *not* exposed to any systematic alignment of formats for rational numbers with types of entities.

Experiment 2

Experiment 2 was designed to determine whether a systematic alignment pattern for rational numbers might emerge for college students, despite the absence of such alignments in formal math instruction in Russia. Specifically, Experiment 2 was designed to replicate Experiment 1 in Rapp et al. (2015) and Experiment 1 from Lee et al. (2015) with Russian college students. If adults have assimilated an alignment between formats for rational numbers and discrete and continuous entities that they encounter in the real world, then perhaps Russian college students will show a pattern of alignment similar to that exhibited by American and South Korean college students, despite the dramatic differences in the way rational numbers are taught in formal math instruction.

Method

Participants

Sixty-four undergraduates (mean age 20 years; 42 females and 22 males) from the Department of Computer Sciences, National Research University Higher School of Economics, were asked to take part in the experiment in lieu of their regular class. They were randomly assigned in equal numbers to one of the two experimental groups.

Procedure

The instructions given to the participants were exactly the same as those used in Experiment 1 from Rapp et al. (2015) and Experiment 1 from Lee et al. (2015). The participants completed the task using paper and pencil. Each of them was given a sheet of paper containing three examples of simple word problems with whole numbers. Two of these included discrete objects (e.g., balls, children) and one included a continuous entity (flour). In order not to influence the alignment between rational numbers and entity type, the sample problems used only whole numbers.

The students were instructed to create two word problems. Half of the students were told that their problems had to contain a fraction (e.g., 1/4, 5/2). The other half of the students were told that their problems had to contain a decimal (e.g., 0.25, 1.3).

Results and discussion

Coding

The constructed problems were coded using criteria based on the study by Rapp et al. (2015). These coding criteria were very similar to those used for the textbook analysis of word problems containing rational numbers. Table 6 shows examples of the problems generated.

The results of Experiment 2 are shown in Table 7. Russian college students more often used continuous entities with decimals (74%) compared to using continuous entities with fractions (51%). Conversely, they used discrete entities more often with fractions (49%) than with decimals (26%). A test of independence between number and object type confirmed that number type and continuity were significantly associated, $\chi^2(1) = 6.76$, $p = 0{,}009$; Phi = 0.224. This pattern of alignment is strikingly similar to that found with

Table 6. Examples of problems generated with different unit types (Experiment 2).

Entity type	Unit type	Example of problem
Continuous	Mass nouns, units of measure (e.g., kg, liter, degrees Celsius)	Artem and Igor bought a bottle of beer. Artem drank $^1/_2$ of this beer and Igor drank $^1/_2$ of remained amount. How much beer did Igor drink if the bottle's volume is 1 liter?
		There were 5 kilos of sugar in a sack. The grandmother took 1.5 kilos to make jam. How much sugar is in a sack now?
Discrete	Collective nouns (class, group, etc.), slices of a mass (pieces of a cake), discrete objects (e.g., a balloon, a stone, etc.)	$^1/_2$ of the whole study group attended a lecture. 1/4 of the group left during a break but 1/3 of the students came after the break. Which part of the whole group was present at the second lecture?
		There were 60 dogs in an animal shelter. 0.33 of them were taken home after a fair. How many dogs have left?

Table 7. Frequencies of alignments between formats for rational numbers and entity types in problems constructed by Russian college students, compared with data for US and South Korean students (Rapp et al., 2015, Experiment 1; Lee et al., 2015, Experiment 2).

	Russia		U.S.		South Korea	
	Decimal	Fraction	Decimal	Fraction	Decimal	Fraction
Continuous	43 (74%)	30 (51%)	94 (72%)	44 (34%)	64 (91%)	46 (63%)
Discrete	15 (26%)	29 (49%)	36 (28%)	86 (66%)	6 (9%)	26 (36%)
N	58 (100%)	59 (100%)	130 (100%)	130 (100%)	70 (100%)	72 (100%)

American students tested by Rapp et al. (2015), as well as by South Korean students tested by Lee et al. (2015).

In summary, Russian college students, like their counterparts in the USA and South Korea, and unlike Russian math textbooks, tend to use decimals to represent continuous entities and fractions to represent discrete entities.

General discussion

The present findings demonstrate a partial dissociation between the pattern of semantic alignments observed in Russia and that previously observed in the United States (and South Korea). In the case of word problems based on natural number arithmetic, Russian educators use arithmetic problems that are consistent with semantic alignment in a manner similar to their use by US educators. Similarly, Russian adults show the same pattern of alignment with addition and division problems as US adults.

However, Russian educators differ in their approach to introducing and teaching rational numbers. Both fraction and decimal problems are taught largely with continuous entities. In fact, only a very small proportion of the total problems in textbooks involve discrete entities (<5%). In general, the approach to teaching fractions and decimals differs from the USA. In Russia, both fractions and decimals are highlighted as tools for continuous measurement. By contrast, in the USA, a stronger emphasis is placed on using decimals as tools of continuous measurement and fractions to represent relations between countable sets. Despite the obvious differences in instruction concerning rational numbers, Russian adult college students showed an alignment pattern for rational numbers that is strikingly similar to that observed with US and South Korean college students. Given that Russian instruction in rational numbers differs so much from US and Korean instruction, the similarity in the performance of college students in these countries indicates that the alignment that we observed for students in Russia cannot be due to formal instruction alone.

A central question that remains to be addressed is what alternative mechanisms may lead to semantic alignments in cases in which formal instruction can be ruled out as the primary cause. Future studies should examine how alignments develop across childhood and into adulthood. It is an open question whether the Russian emphasis on continuous measurement, irrespective of rational number format, has any effect on student understanding of fractions and decimals. Some previous research (e.g., Moss & Case, 1999) has shown that introducing percentages and decimals with an emphasis on measurement before introducing fractions can improve general understanding of fractions' magnitudes. The Russian curriculum may provide an advantage for magnitude assessments of rational numbers – especially those of fractions, with which US and Korean students struggle more than with decimals (e.g., DeWolf et al., 2015; Lee et al., 2015). In contrast, the American emphasis on discrete entities may better support understanding of relational reasoning with fractions (e.g., DeWolf et al., 2015), since a fraction represents the relation between the cardinalities of two sets.

A question for instructional research is whether explicit instruction in semantic alignments would impact the later use of such alignments by students. In all the curricula so far investigated, even when textbook examples exhibit alignments, this fact is never explicitly stated or highlighted to students. Nonetheless, it seems that even when an alignment is *not* present in textbooks, as in the Russian curriculum for rational numbers, students ultimately make use of the alignment. However, studies to date, including the present one, have only measured implicit sensitivity to alignment. It is, therefore, unclear to what extent students and textbook writers are aware of the alignment.

A basic question is whether acquiring semantic alignments is in fact desirable. If semantic alignment is a heuristic strategy for interpreting situation models in terms of mathematical models, as Bassok et al. (1998) proposed, students can benefit from such alignments when solving problems and constructing equations (Fisher et al., 2011; Martin & Bassok, 2005). However, it remains to be established whether there are any negative implications of semantic alignments, and to consider whether the benefits of the heuristic outweigh its possible negative consequences. The central goal of Russian math instruction is to move the student quickly from a focus on concrete entities to a focus on abstract mathematical concepts. But despite this educational aim, Russian adults (much like Americans) exhibit a clear influence of semantic alignments. It seems that people (at least those who do not become mathematicians) have a propensity to anchor mathematical abstractions in properties of concrete entities.

One educational strategy that deserves to be explored would be to explicitly teach semantic alignments, while also calling attention to the ways in which mathematical concepts ultimately transcend them. For example, it may well be a useful heuristic to favour applying fractions to discrete quantities and decimals to continuous ones, even though it is also important to understand that both formats for rational numbers may be used with both quantity types. Semantic alignments for mathematical expressions may function much like prototypes for natural categories, providing easy-to-process examples that facilitate understanding of the relations they instantiate (cf. Murphy, 2002). But just as people have to learn that natural categories can include a broad range of examples, it is also essential to grasp that a mathematical concept can be instantiated by a wide range of entities, not all of which will be “prototypical”. The educational goal should be to make semantic alignments available as useful heuristics that facilitate mathematical thinking in many common situations, while ensuring they do not artificially constrain the concrete entities that a mathematical concept can describe.

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References

Bassok, M., & Olseth, K. L. (1995). Object-based representations: Transfer between cases of continuous and discrete models of change. *Journal of Experimental Psychology: Learning Memory and Cognition*, *21*(6), 354–367.

Bassok, M., Chase, V. M., & Martin, S. A. (1998). Adding apples and oranges: Alignment of semantic and formal knowledge. *Cognitive Psychology*, *35*(2), 99–134.

Bassok, M., Pedigo, S. F., & Oskarsson, A. T. (2008). Priming addition facts with semantic relations. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *34* (2), 343–352.

Bassok, M., Wu, L., & Olseth, K. L. (1995). Judging a book by its cover: Interpretative effects of context on problem-solving transfer. *Memory & Cognition*, *23*, 354–367.

Bourbaki, N. (1950). The architecture of mathematics. *American Mathematical Monthly*, *57*, 221–232.

Brown, L. D., Cai, T., & Dagupta, A. (2001). Interval estimation for a binomial proportion. *Statistical Science*, *16*, 101–133.

Bruner, J. (1990). *Acts of meaning*. Cambridge, MA: Harvard University Press.

Corry, L. (1992). Nicolas Bourbaki and the concept of mathematical structure. *Synthese*, *92*, 315–348.

Davydov, V. V. (1982). The psychological characteristics of the formation of elementary mathematical operations in children. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 224–238). Hillsdale, NJ: Erlbaum.

Davydov, V. V. (1990). Types of generalization in instruction: Logical and psychological problems in the structuring of school curricula. *Soviet Studies in Mathematics Education*, Vol. 2. Reston, VA: National Council of Teachers of Mathematics.

DeWolf, M., Bassok, M., & Holyoak, K. J. (2015). Conceptual structure and the procedural affordances of rational numbers: Relational reasoning with fractions and decimals. *Journal of Experimental Psychology: General*, *144*(1), 127–150.

Demidova, M. Y., Kovaleva, G. S., Koshelenko, N. G., Krasnyanskaya, K. A., Loginova, O. B., & Smirnova, E. S. (2013). *Osnovnye rezultaty mezhdunarodnogo issledovaniya kachestva matematicheskogo i estestvennonauchnogo obrazovaniya TIMSS-2011. Analiticheskij otchet.* [Main results of the international assessment of the mathematics and science knowledge TIMSS-2011. Analytic report]. Moscow: MAKS Press.

Dixon, J. A., Deets, J. K., & Bangert, A. (2001). The representations of the arithmetic operations include functional relationships. *Memory & Cognition*, *29*(3), 462–477.

Dorofeev, G., & Peterson, L. (2011). *Matematika, 5 klass* [Mathematics, 5th grade] (2th ed.). Moscow: Mnemozina. Russian.

Fisher, K. J., Borchert, K., & Bassok, M. (2011). Following the standard form: Effects of equation format on algebraic modeling. *Memory & Cognition*, *39*(3), 502–515.

Guthormsen, A. M., Fisher, K. J., Bassok, M., Osterhout, L., DeWolf, M., & Holyoak, K. J. (2016). Conceptual integration of arithmetic operations with real-world knowledge: Evidence from event-related potentials. *Cognitive Science*, *40*, 723–757.

Holyoak, K. J. (2012). Analogy and relational reasoning. In K. J. Holyoak & R. G. Morrison (Eds.), *The Oxford handbook of thinking and reasoning* (pp. 234–259). New York, NY: Oxford University Press.

Kelly, A., & Washtell, A. (1996). "Developmental education" in Russia. *Early Child Development and Care*, *124*, 81–89.

Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review*, *92*(1), 109–129.

Kolmogorov, A. N. (1990). K obsuzhdeniju raboty po probleme «Perspektivy razvitija sovetskoj shkoly na blizhajshie tridcat' let» [To a discussion of the problem "Perspectives of development of Soviet school for the next 30 years"]. *Matematika v shkole [Mathematics in School]*, *5*, 9–61 . Russian.

Lamon, S. (1993). Ratio and proportion: Children's cognitive and metacognitive processes. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 131–156). Hillsdale, NJ: Erlbaum.

Lee, H. S., DeWolf, M., Bassok, M., & Holyoak, K. J. (2015). Conceptual and procedural distinctions between fractions and decimals: A cross-national comparison. *Cognition*, *147*, 57–69.

Makarychev, J. N., Mindjuk, N. G., Meshkov, K. I., & Feoktistov, I. E. (2008). *Algebra, 7 klass* [Algebra, 7th grade] (17th ed.). Moscow: Prosveshchenie.

Martin, S. A., & Bassok, M. (2005). Effects of semantic cues on mathematical modeling: Evidence from word-problem solving and equation construction tasks. *Memory & Cognition*, *33*(3), 471–478.

Ministry of Education and Science of the Russian Federation. (2004). Federalny gosudarstvenny obrazovatelny standart nachalnogo obtsego obrazovanija [Federal state education standard of the primary general education]. Retrieved from http://mosmetod.ru/metodicheskoe-prostranstvo/nachalnaya-shkola/fgos/fgos-noo-sizmeneniyami-na-18-maya-2015-goda.html

Ministry of Education and Science of the Russian Federation. (2011). Primernaya osnovnaya obrazovatlnaya programma obrazovatelnogo uchrejdeniya: Nachalnaya shkola [Model basic educational program of the educational institution: Primary school]. Retrieved from http://fgosreestr.ru/registry/primernaya-osnovnayaobrazovatelnaya-programma-nachalnogo-obshhego-obrazovaniya-2/5

Moro, M. I., Bantova, M. A., Beltyukova, G. V., Volkova, S. I., & Stepanova, S. V. (2011). *Matematika, 4 klass* [Mathematics, 4th grade] (8th ed.). Moscow: Prosveshchenie.

Moro, M. I., Volkova, S. I., Stepanova, S. V., Bantova, M. A., & Beltyukova, G. V. (2016). Matematika. Rabochie programmy. Predmetnaya liniya uchebnikov sistemy "Shkola Rossii." 1-4-klassy: Ucheb. posobie dlya obshcheobrazovat. organizacij [Mathematics. Working programs. Subject line of textbooks for system "Schools of Russia." 1-4 grades: Tutorial for general school] (2nd ed.). Moscow: Prosveshchenie.

Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for Research in Mathematics Education*, *30*, 122.

Mullis, I. V. S., Minnich, C. A., Arora, G. M., Centurino, V. A. S., & Castle, C. E. (2012). *TIMSS 2011 Encyclopedia: Education Policy and Curriculum in Mathematics and Science*. Vol. 2, L-Z and benchmarking participants. Chestnu Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Murphy, G. L. (2002). *The big book of concepts*. Cambridge, MA: MIT Press.

Piaget, J. (1970). *The science of education and the psychology of the child*. New York, NY: Grossman.

Rapp, M., Bassok, M., DeWolf, M., & Holyoak, K. J. (2015). Modeling discrete and continuous entities with fractions and decimals. *Journal of Experimental Psychology: Applied*, *21*(1), 47–56.

Roslova, L. O. (2009). Metodika prepodavaniya naglyadnoj geometrii uchashchimsya 5-6 klassov. Lekciya 1 [Instructional design for visual geometry in the 5-6th grades. Lecture 1]. *Matematika [Mathematics]*, *17*(679). Retrieved from http://mat.1september.ru/view_article.php?ID=200901711

Sophian, C. (2004). Mathematics for the future: Developing a Head Start curriculum to support mathematics learning. *Early Childhood Research Quarterly*, *19*, 59–81.

Vilenkin, N. Y., & Zhokhov, V. I. (2008). *Matematika, 6 klass* [Mathematics, 6th grade] (23th ed.). Moscow: Mnemozina.

Vilenkin, N. Y., Zhokhov, V. I., & Schwartzburd, S. I. (2005). *Matematika, 5 klass* [Mathematics, 5th grade] (17th ed.). Moscow: Mnemozina.

Vilenkin, N. Y., Zhokhov, V. I., & Schwartzburd, S. I. (2013). *Matematika, 5 klass* [Mathematics, 5th grade] (31th ed.). Moscow: Mnemozina.

Vilenkin, N. Y., Zhokhv, V. I., & Schwartzburd, S. I. (2008). Matematika, 5 klass [Mathematics, 5th grade] (24th ed.). Moscow: Mnemozina.

Zubareva, I., & Mordkovich, A. (2013). *Matematika, 5 klass* [Mathematics, 5th grade] (14th ed.). Moscow: Mnemozina.