CHAPTER 6

Analogy

Keith J. Holyoak

Analogy is a special kind of similarity (see Goldstone & Son, Chap. 2). Two situations are analogous if they share a common pattern of relationships among their constituent elements even though the elements themselves differ across the two situations. Typically, one analog, termed the *source* or *base*, is more familiar or better understood than the second analog, termed the *target*. This asymmetry in initial knowledge provides the basis for analogical transfer, using the source to generate inferences about the target. For example, Charles Darwin drew an analogy between breeding programs used in agriculture to select more desirable plants and animals and "natural selection" for new species. The well-understood source analog called attention to the importance of variability in the population as the basis for change in the distribution of traits over successive generations and raised a critical question about the target analog: What plays the role of the farmer in natural selection? (Another analogy, between Malthus' theory of human population growth and the competition of individuals in a species to survive and reproduce, provided Darwin's answer to this question.) Analogies have figured prominently in the history of science (see Dunbar & Fugelsang, Chap. 29) and mathematics (Pask, 2003) and are of general use in problem solving (see Novick & Bassok, Chap. 14). In legal reasoning, the use of relevant past cases (legal precedents) to help decide a new case is a formalized application of analogical reasoning (see Ellsworth, Chap. 28). Analogies can also function to influence political beliefs (Blanchette & Dunbar, 2001) and to sway emotions (Thagard & Shelley, 2001). Analogical reasoning goes beyond the information initially given, using systematic connections between the source and target to generate plausible, although fallible, inferences about the target. Analogy is thus a form of inductive reasoning (see Sloman & Lagnado, Chap. 5).

Figure 6.1 sketches the major component processes in analogical transfer (see Carbonell, 1983; Gentner, 1983; Gick & Holyoak, 1980, 1983; Novick & Holyoak, 1991). Typically, a target situation serves as a retrieval cue for a potentially useful source analog. It is then necessary to establish a *mapping*, or a set of systematic correspondences that serve to align the

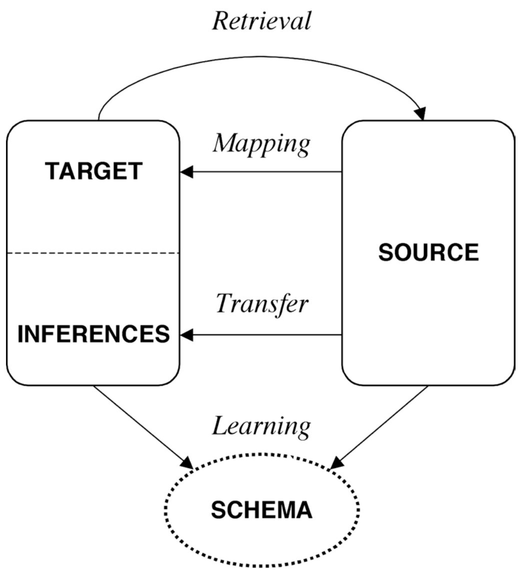

Figure 6.1. Major components of analogical reasoning.

elements of the source and target. On the basis of the mapping, it is possible to derive new inferences about the target, thereby elaborating its representation. In the aftermath of analogical reasoning about a pair of cases, it is possible that some form of relational generalization may take place, yielding a more abstract schema for a class of situations, of which the source and target are both instances. For example, Darwin's use of analogy to construct a theory of natural selection ultimately led to the generation of a more abstract schema for a selection theory, which in turn helped to generate new specific theories in many fields, including economics, genetics, sociobiology, and artificial intelligence. Analogy is one mechanism for effecting conceptual change (see Chi & Ohlsson, Chap. 16).

A Capsule History

The history of the study of analogy includes three interwoven streams of research, which respectively emphasize analogy in relation to psychometric measurement of intelligence, metaphor, and the representation of knowledge.

Psychometric Tradition

Work in the psychometric tradition focuses on four-term or "proportional" analogies in the form A:B::C:D, such as HAND: FINGER :: FOOT: ?, where the problem is to infer the missing D term (TOE) that is related to C in the same way B is related to A (see Sternberg, Chap. 31). Thus A:B plays the role of source analog and C:D plays the role of target. Proportional analogies were discussed by Aristotle (see Hesse, 1966) and in the early decades of modern psychology became a centerpiece of efforts to define and measure intelligence. Charles Spearman (1923, 1927) argued that the best account of observed individual differences in cognitive performance was based on a general or *g* factor, with the remaining variance being unique to the particular task. He reviewed several studies that revealed high correlations between performance in solving analogy problems and the *g* factor. Spearman's student John C. Raven (1938) developed the Raven's Progressive Matrices Test (RPM), which requires selection of a geometric figure to fill an empty cell in a two-dimensional matrix (typically 3×3) of such figures. Similar to a geometric proportional analogy, the RPM requires participants to extract and apply information based on visuospatial relations. (See Hunt, 1974, and Carpenter, Just, & Shell, 1990, for analyses of strategies for solving RPM problems.) The RPM proved to be an especially pure measure of *g*.

Raymond Cattell (1971), another student of Spearman, elaborated his mentor's theory by distinguishing between two components of *g*: *crystallized* intelligence, which depends on previously learned information or skills, and *fluid* intelligence, which involves reasoning with novel information. As a form of inductive reasoning, analogy would be expected to require fluid intelligence. Cattell confirmed Spearman's (1946) observation that analogy tests and the RPM provide sensitive measures of *g*, clarifying that

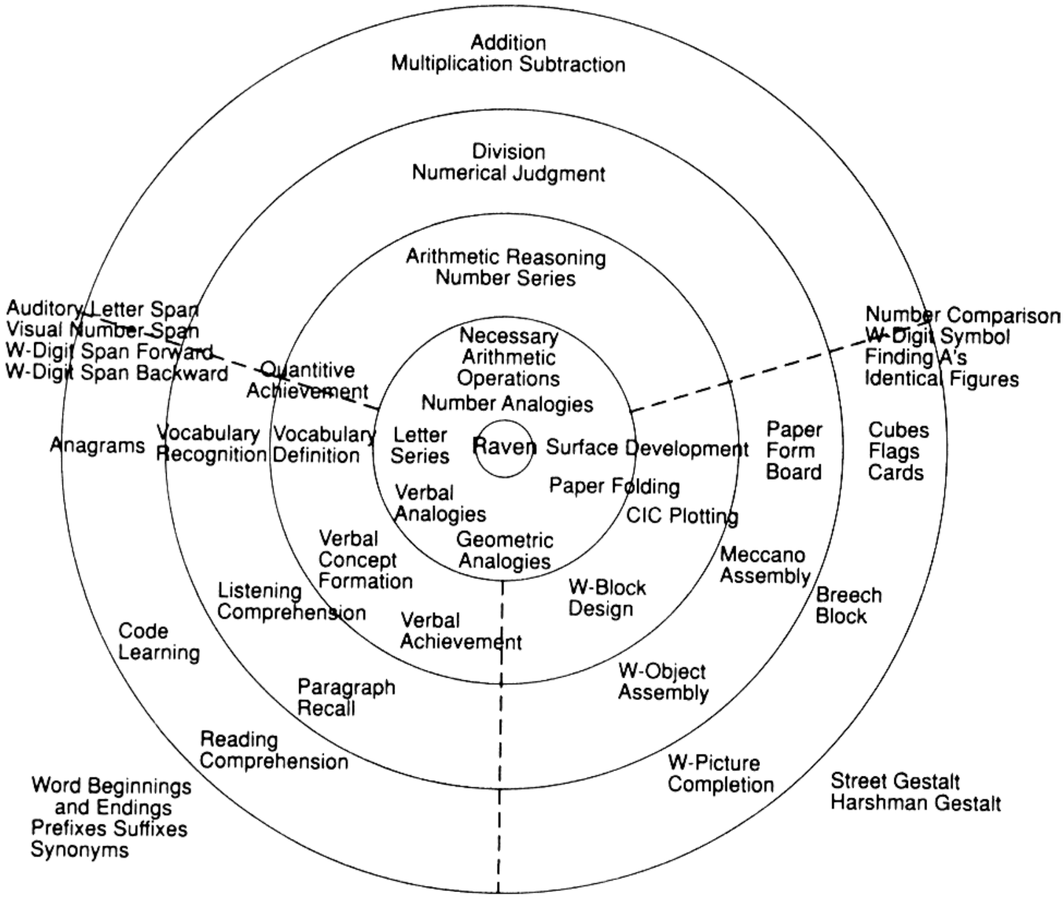

Figure 6.2. Multidimensional scaling solution based on intercorrelations among the Raven's Progressive Matrices test, analogy tests, and other common tests of cognitive function. (From Snow, Kyllonen, & Marshalek, 1984, p. 92. Reprinted by permission.)

they primarily measure fluid intelligence (although verbal analogies based on difficult vocabulary items also depend on crystallized intelligence). Figure 6.2 graphically depicts the centrality of RPM performance in a space defined by individual differences in performance on various cognitive tasks. Note that numeric, verbal, and geometric analogies cluster around the RPM at the center of the figure.

Because four-term analogies and the RPM are based on small numbers of relatively well-specified elements and relations, it is possible to manipulate the complexity of such problems systematically and analyze performance (based on response latencies and error rates) in terms of component processes (e.g., Mulholland, Pellegrino, & Glaser, 1980; Sternberg, 1977). The earliest computational models of analogy were developed for four-term analogy problems (Evans, 1968; Reitman, 1965). The basic components of these models were elaborations of those proposed by Spearman (1923), including encoding of the terms, accessing a relation between the A and B terms, and evoking a comparable relation between the C and D terms.

More recently, four-term analogy problems and the RPM have figured prominently in neuropsychological and neuroimaging studies of reasoning (e.g., Bunge, Wendelken, Badre & Wagner, 2004; Kroger et al., 2002; Luo et al., 2003; Prabhakaran et al., 1997; Waltz et al., 1999; Wharton et al., 2000). Analogical reasoning depends on working memory (see Morrison, Chap. 19). The neural basis of working memory includes the dorsolateral prefrontal cortex, an area of the brain that becomes increasingly activated as the complexity of the problem (measured in terms of number of relations relevant to the solution) increases. It has been argued that this area underlies the fluid component of Spearman's *g* factor in intelligence (Duncan et al., 2000), and it plays an important role in many reasoning tasks (see Goel, Chap. 20).

Metaphor

Analogy is closely related to metaphor and related forms of symbolic expression that arise in everyday language (e.g., "the evening of life," "the idea blossomed"), in literature (Holyoak, 1982), the arts, and cultural practices such as ceremonies (see Holyoak & Thagard, 1995, Chap. 9). Similar to analogy in general, metaphors are characterized by an asymmetry between target (conventionally termed "tenor") and source ("vehicle") domains (e.g., the target/tenor in "the evening of life" is life, which is understood in terms of the source/vehicle of time of day). In addition, a mapping (the "grounds" for the metaphor) connects the source and target, allowing the domains to interact to generate a new conceptualization (Black, 1962). Metaphors are a special kind of analogy in that the source and target domains are always semantically distant (Gentner, 1982; Gentner, Falkenhainer, & Skorstad, 1988), and the two domains are often blended rather than simply mapped (e.g., in "the idea blossomed," the target is directly described in terms of an action term derived from the source). In addition, metaphors are often combined with other symbolic "figures" – especially metonymy (substitution of an associated concept). For example, "sword" is a metonymic expression for weaponry, derived from its ancient association as the prototypical weapon – "Raising interests rates is the Federal Reserve Board's sword in the battle against inflation" extends the metonymy into metaphor.

Fauconnier and Turner (1998; Fauconnier, 2001) analyzed complex conceptual blends that are akin to metaphor. A typical example is a description of the voyage of a modern catamaran sailing from San Francisco to Boston that was attempting to beat the speed record set by a clipper ship that had sailed the same route over a century earlier. A magazine account written during the catamaran's voyage said the modern boat was "barely maintaining a 4.5 day lead over the ghost of the clipper *Northern Light*. . . . " Fauconnier and Turner observed that the magazine writer was describing a "boat race" that never took place in any direct sense; rather, the writer was blending the separate voyages of the two ships into an imaginary race. The fact that such conceptual blends are so natural and easy to understand attests to the fact that people can readily comprehend novel metaphors.

Lakoff and Johnson (1980; also Lakoff & Turner, 1989) argued that much of human experience, especially its abstract aspects, is grasped in terms of broad conceptual metaphors (e.g., events occurring in time are understood by analogy to objects moving in space). Time, for example, is understood in terms of objects in motion through space as in expressions such as "My birthday is fast approaching" and "The time for action has arrived." (See Boroditsky, 2000, for evidence of how temporal metaphors influence cognitive judgments.) As Lakoff and Turner (1989) pointed out, the course of a life is understood in terms of time in the solar year (youth is springtime; old age is winter). Life is also conventionally conceptualized as a journey. Such conventional metaphors can still be used in creative ways, as illustrated by Robert Frost's famous poem, "The Road Not Taken":

> Two roads diverged in a wood, and I –
> I took the one less traveled by,
> And that has made all the difference.

According to Lakoff and Turner, comprehension of this passage depends on our implicit knowledge of the metaphor that life is a journey. This knowledge includes understanding several interrelated correspondences (e.g., person is a traveler, purposes are destinations, actions are routes, difficulties in life are impediments to travel, counselors are guides, and progress is the distance traveled).

Psychological research has focused on demonstrations that metaphors are integral to everyday language understanding (Glucksberg, Gildea, & Bookin, 1982; Keysar, 1989) and debate about whether metaphor is better conceptualized as a kind of analogy (Wolff & Gentner, 2000) or a kind of categorization (Glucksberg & Keysar, 1990; Glucksberg, McClone, & Manfredi, 1997). A likely resolution is that novel metaphors are interpreted by much the same process as analogies, whereas more conventional metaphors are interpreted as more general schemas (Gentner, Bowdle, Wolff, & Boronat, 2001).

Knowledge Representation

The most important influence on analogy research in the cognitive science tradition has been concerned with the representation of knowledge within computational systems. Many seminal ideas were developed by the philosopher Mary Hesse (1966), who was in turn influenced by Aristotle's discussions of analogy in scientific classification and Black's (1962) interactionist view of metaphor. Hesse placed great stress on the purpose of analogy as a tool for scientific discovery and conceptual change and on the close connections between causal relations and analogical mapping. In the 1970s, work in artificial intelligence and psychology focused on the representation of complex knowledge of the sort used in scientific reasoning, problem solving, story comprehension, and other tasks that require structured knowledge. A key aspect of structured knowledge is that elements can be flexibly *bound* into the roles of relations. For example, "dog bit man" and "man bit dog" have the same elements and the same relation, but the role bindings have been reversed, radically altering the meaning. How the mind and brain accomplish role binding is thus a central problem to be solved by any psychological theory of structured knowledge, including any theory of analogy (see Doumas & Hummel, Chap. 4).

In the 1980s, a number of cognitive scientists recognized the centrality of analogy as a tool for discovery and its close connection with theories of knowledge representation. Winston (1980), guided by Minsky's (1975) treatment of knowledge representation, built a computer model of analogy that highlighted the importance of causal relations in guiding analogical inference. Other researchers in artificial intelligence also began to consider the use of complex analogies in reasoning and learning (Kolodner, 1983; Schank, 1982), leading to an approach to artificial intelligence termed *case-based reasoning* (see Kolodner, 1993).

Around 1980, two research projects in psychology began to consider analogy in relation to knowledge representation and eventually integrate computational modeling with detailed experimental studies of human analogical reasoning. Gentner (1982, 1983; Gentner & Gentner, 1983) began working on mental models and analogy in science. She emphasized that in analogy, the key similarities lie in the *relations* that hold within the domains (e.g., the flow of electrons in an electrical circuit is analogically similar to the flow of people in a crowded subway tunnel), rather than in features of individual objects (e.g., electrons do not resemble people). Moreover, analogical similarities often depend on *higher-order* relations – relations *between* relations. For example, adding a resistor to a circuit *causes* a decrease in flow of electricity, just as adding a narrow gate in the subway tunnel would decrease the rate at which people pass through (where *causes* is a higher-order relation). In her structure-mapping theory, Gentner proposed that analogy entails finding a structural alignment, or mapping, between domains. In this theory, alignment between two representational structures is characterized by structural parallelism (consistent, one-to-one correspondences between mapped elements) and systematicity – an implicit preference for deep, interconnected systems of relations governed by higher-order relations, such as causal, mathematical, or functional relations.

Holyoak (1985; Gick & Holyoak, 1980, 1983; Holyoak & Koh, 1987) focused on the role of analogy in problem solving with a strong concern for the role of pragmatics in analogy – that is, how causal relations that impact current goals and context guide the interpretation of an analogy. Holyoak and Thagard (1989a, 1995) developed an approach to analogy in which several factors were viewed as jointly constraining analogical reasoning. According to their multiconstraint theory, people tend to find mappings that maximize *similarity* of corresponding elements and relations, *structural* parallelism (i.e., isomorphism, defined by consistent, one-to-one correspondences), and *pragmatic* factors such as the importance of elements and relations for achieving a goal. Gick and Holyoak (1983) provided evidence that analogy can furnish the seed for forming new relational categories by abstracting the relational correspondences between examples into a schema for a class of problems. Analogy was viewed as a central part of human induction (Holland, Holyoak, Nisbett, & Thagard, 1986; see Sloman & Lagnado, Chap. 5) with close ties to other basic thinking processes, including causal inference (see Buehner & Cheng, Chap. 7), categorization (see Medin & Rips, Chap. 3), deductive reasoning (see Evans, Chap. 8), and problem solving (see Novick & Bassok, Chap. 14).

Analogical Reasoning: Overview of Phenomena

This section provides an overview of the major phenomena involving analogical reasoning that have been established by empirical investigations. This review is organized around the major components of analogy depicted in Figure 6.1. These components are inherently interrelated, so the connections among them are also discussed. The retrieval and mapping components are first considered followed by inference and relational generalization.

Retrieval and Mapping

A PARADIGM FOR INVESTIGATING ANALOGICAL TRANSFER

Gick and Holyoak (1980, 1983) introduced a general laboratory paradigm for investigating analogical transfer in the context of problem solving. The general approach was first to provide people with a source analog in the guise of some incidental context, such as an experiment on "story memory." Later, participants were asked to solve a problem that was in fact analogous to the story they had studied earlier. The questions of central interest were (1) whether people would spontaneously notice the relevance of the source analog and use it to solve the target problem, and (2) whether they could solve the analogy once they were cued to consider the source. Spontaneous transfer of the analogous solution implies successful retrieval and mapping; cued transfer implies successful mapping once the need to retrieve the source has been removed.

The source analog used by Gick and Holyoak (1980) was a story about a general who is trying to capture a fortress controlled by a dictator and needs to get his army to the fortress at full strength. Because the entire army could not pass safely along any single road, the general sends his men in small groups down several roads simultaneously. Arriving at the same time, the groups join together and capture the fortress.

A few minutes after reading this story under instructions to read and remember it (along with two other irrelevant stories), participants were asked to solve a tumor problem (Duncker, 1945), in which a doctor has to figure out how to use rays to destroy a stomach tumor without injuring the patient in the process. The crux of the problem is that it seems that the rays will have the same effect on the healthy tissue as on the tumor – high intensity will destroy both, whereas low intensity will destroy neither. The key issue is to determine how the rays can be made to impact the tumor selectively while sparing the surrounding tissue. The source analog, if it can be retrieved and mapped, can be used to generate a "convergence" solution to the tumor problem, one that parallels the general's military strategy: Instead of using a single high-intensity ray, the doctor could administer several low-intensity rays at once from different directions. In that way, each ray would be at low intensity along its path, and hence, harmless to the healthy tissue, but the effects of the rays would sum to achieve the effect of a high-intensity ray at their focal point, the site of the tumor.

When Gick and Holyoak (1980) asked college students to solve the tumor problem, without a source analog, only about 10% of them produced the convergence solution. When the general story had been studied, but no hint to use it was given, only about 20% of participants produced the convergence solution. In contrast, when the same participants were then given a simple hint that "you may find one of the stories you read earlier to be helpful in solving the problem," about 75% succeeded in generating the analogous convergence solution. In other words, people often fail to notice superficially dissimilar source analogs that they could readily use.

This gap between the difficulty of retrieving remote analogs and the relative ease of mapping them has been replicated many times, both with adults (Gentner, Rattermann, & Forbus, 1993; Holyoak & Koh, 1987; Spencer & Weisberg, 1986) and with young children (Chen, 1996; Holyoak, Junn, & Billman, 1984; Tunteler & Resing, 2002). When analogs must be cued from long-term memory, cases from a domain similar to that of the cue are retrieved much more readily than cases from remote domains (Keane, 1987; Seifert, McKoon, Abelson, & Ratcliff, 1986). For example, Keane (1987) measured retrieval of a convergence analog to the tumor problem when the source analog was studied 1 to 3 days prior to presentation of the target radiation problem. Keane found that 88% of participants retrieved a source analog from the same domain (a story about a surgeon treating a brain tumor), whereas only 12% retrieved a source from a remote domain (the general story). This difference in ease of access was dissociable from the ease of postaccess mapping and transfer because the frequency of generating the convergence solution to the radiation problem once the source analog was cued was high and equal (about 86%), regardless of whether the source analog was from the same or a different domain.

DIFFERENTIAL IMPACT OF SIMILARITY AND STRUCTURE ON RETRIEVAL VERSUS MAPPING

The main empirical generalization concerning retrieval and mapping is that similarity of individual concepts in the analogs has a relatively greater impact on retrieval, whereas mapping is relatively more sensitive to relational correspondences (Gentner et al., 1993; Holyoak & Koh, 1987; Ross, 1987, 1989). However, this dissociation is not absolute. Watching the movie *West Side Story* for the first time is likely to trigger a reminding of Shakespeare's *Romeo and Juliet* despite the displacement of the characters in the two works over centuries and continents. The two stories both involve young lovers who suffer because of the disapproval of their respective social groups, causing a false report of death, which in turn leads to tragedy. It is these structural parallels between the two stories that make them analogous rather than simply that both stories involve a young man and woman, a disapproval, a false report, and a tragedy.

Experimental work on story reminding confirms the importance of structure, as well as similarity of concepts, in retrieving analogs from memory. Wharton and his colleagues (Wharton et al., 1994; Wharton, Holyoak, & Lange, 1996) performed a series of experiments in which college students tried to find connections between stories that overlapped in various ways in terms of the actors and actions and the underlying themes. In a typical experiment, the students first studied about a dozen "target" stories presented in the guise of a study of story understanding. For example, one target story exemplified a theme often called "sour grapes" after one of Aesop's fables. The theme in this story is that the protagonist tries to achieve a goal, fails, and then retroactively decides the goal had not really been desirable after all. More specifically, the actions involved someone trying unsuccessfully to get accepted to an Ivy League college. After a delay, the students read a set of different cue stories and were asked to write down any story or stories from the first session of which they were reminded. Some stories (far analogs) exemplified the same theme, but with very different characters and actions (e.g., a "sour grapes" fairy tale about a unicorn who tries to cross a river but is forced to turn back). Other stories were far "disanalogs" formed by reorganizing the characters and actions to represent a distinctly different theme (e.g., "self-doubt" – the failure to achieve a goal leads the protagonist to doubt his or her own ability or merit). Thus, neither type of cue was similar to the target story in terms of individual elements (characters and actions); however, the far analog maintained structural correspondences of higher-order causal relations with the target story, whereas the far disanalog did not.

Besides varying the relation between the cue and target stories, Wharton et al. (1994) also varied the number of target stories that were in some way related to a single cue. When only one target story in a set had been studied ("singleton" condition), the probability of reminding was about equal, regardless of whether the cue was analogous to the target. However, when two target stories had been studied (e.g., both "sour grapes" and "self-doubt," forming a "competition" condition), the analogous target was more likely to be retrieved than the disanalogous one. The advantage of the far analog in the competition condition was maintained even when a week intervened between initial study of the target stories and presentation of the cue stories (Wharton et al., 1996).

These results demonstrate that structure does influence analogical retrieval, but its impact is much more evident when multiple memory traces, each somewhat similar to the cue, must compete to be retrieved. Such retrieval competition is likely typical of everyday analogical reminding. Other evidence indicates that having people generate case examples, as opposed to simply asking them to remember cases presented earlier, enhances structure-based access to source analogs (Blanchette & Dunbar, 2000).

THE "RELATIONAL SHIFT" IN DEVELOPMENT

Retrieval is thus sensitive to structure and direct similarity of concepts. Conversely, mapping is sensitive to direct similarity and structure (e.g., Reed, 1987; Ross, 1989). Young children are particularly sensitive to direct similarity of objects; when asked to identify corresponding elements in two analogs, their mappings are dominated by object similarity when semantic and structural constraints conflict (Gentner & Toupin, 1986). Younger children are particularly likely to map on the basis of object similarity when the relational response requires integration of multiple relations, and hence, is more dependent on working memory resources (Richland, Morrison, & Holyoak, 2004). The developmental transition toward greater reliance on structure in mapping has been termed the "relational shift" (Gentner & Rattermann, 1991). Greater sensitivity to relations with age appears to arise owing to a combination of incremental accretion of knowledge about relational concepts and stage-like increments in working memory capacity (Halford, 1993; Halford & Wilson, 1980). (For reviews of developmental research on analogy, see Goswami, 1992, 2001; Halford, Chap. 22 ; Holyoak & Thagard, 1995).

GOAL-DIRECTED MAPPING

Mapping is guided not only by relational structure and element similarity but also by the goals of the analogist (Holyoak, 1985). People draw analogies not to find a pristine isomorphism for its own sake but to make plausible inferences that will achieve their goals. Particularly when the mapping is inherently ambiguous, the constraint of pragmatic centrality – relevance to goals – is critical (Holyoak, 1985). Spellman and Holyoak (1996) investigated the impact of processing goals on the mappings generated for inherently ambiguous analogies. In one experiment, college students read two science fiction stories about countries on two planets. These countries were interrelated by various economic and military alliances. Participants first made judgments about individual countries based on either economic or military relationships and were then asked mapping questions about which countries on one planet corresponded to which on the other. Schematically, planet 1 included three countries, such that "Afflu" was economically richer than "Barebrute," whereas the latter was militarily stronger than "Compak." Planet 2 included four countries, with "Grainwell" being richer than "Hungerall" and "Millpower" being stronger than "Mightless." The critical aspect of this analogy problem is that Barebrute (planet 1) is both economically weak (like Hungerall on planet 2) and militarily strong (like Millpower) and therefore, has two competing mappings that are equally supported by structural and similarity constraints.

Spellman and Holyoak (1996) found that participants whose processing goal led them to focus on economic relationships tended to map Barebrute to Hungerall rather than Millpower, whereas those whose processing goal led them to focus on military relationships had the opposite preferred mapping. The variation in pragmatic centrality of the information thus served to decide between the competing mappings. One interpretation of such findings is that pragmatically central propositions tend to be considered earlier and more often than those that are less goal relevant and hence, dominate the mapping process (Hummel & Holyoak, 1997).

COHERENCE IN ANALOGICAL MAPPING

The key idea of Holyoak and Thagard's (1989a) multiconstraint theory of analogy is that several different kinds of constraints – similarity, structure, and purpose – all interact to determine the optimal set of correspondences between source and target. A good analogy is one that appears *coherent* in the sense that multiple constraints converge on a solution that satisfies as many different constraints as possible (Thagard, 2000). Everyday use of analogies depends on the human ability to find coherent mappings – even when source and target are complex and the mappings are ambiguous. For example, political debate often makes use of analogies between prior situations and some current controversy (Blanchette & Dunbar, 2001, 2002). Ever since World War II, politicians in the United States and elsewhere have periodically argued that some military intervention was justified because the current situation was analogous to that leading to World War II. A commonsensical mental representation of World War II, the source analog, amounts to a story figuring an evil villain, Hitler; misguided appeasers, such as Neville Chamberlain; and clear-sighted heroes, such as Winston Churchill and Franklin Delano Roosevelt. The countries involved in World War II included the villains, Germany and Japan; the victims, such as Austria, Czechoslovakia, and Poland; and the heroic defenders, notably Britain and the United States.

A series of American presidents have used the World War II analog as part of their argument for American military intervention abroad (see Khong, 1992). These include Harry Truman (Korea, 1950), Lyndon Johnson (Vietnam, 1965), George Bush senior (Kuwait and Iraq, 1991), and his son George W. Bush (Iraq, 2003). Analogies to World War II have also been used to support less aggressive responses. Most notably, during the Cuban missile crisis of 1962, President John F. Kennedy decided against a surprise attack on Cuba in part because he did not want the United States to behave in a way that could be equated to Japan's surprise attack on Pearl Harbor.

The World War II situation was, of course, very complex and is never likely to map perfectly onto any new foreign policy problem. Nonetheless, by selectively focusing on goal-relevant aspects of the source and target and using multiple constraints in combination, people can often find coherent mappings in situations of this sort. After the Iraqi invasion of Kuwait in 1990, President George H. W. Bush argued that Saddam Hussein, the Iraqi leader, was analogous to Adolf Hitler and that the Persian Gulf crisis in general was analogous to events that had led to World War II a half-century earlier. By drawing the analogy between Hussein and Hitler, President Bush encouraged a reasoning process that led to the construction of a coherent system of roles for the players in the Gulf situation. The popular understanding of World War II provided the source, and analogical mapping imposed a set of roles on the target Gulf situation by selectively emphasizing the most salient relational parallels between the two situations. Once the analogical correspondences were established (with Iraq identified as an expansionist dictatorship like Germany, Kuwait as its first victim, Saudi Arabia as the next potential victim, and the United States as the main defender of the Gulf states), the clear analogical inference was that both self-interest and moral considerations required immediate military intervention by the United States. Aspects of the Persian Gulf situation that did not map well to World War II (e.g., lack of democracy in Kuwait) were pushed to the background.

Of course, the analogy between the two situations was by no means perfect. Similarity at the object level favored mapping the United States of 1991 to the United States of World War II simply because it was the same country, which would in turn support mapping Bush to President Roosevelt. However, the United States did not enter World War II until it was bombed by Japan, well after Hitler had marched through much of Europe. One might therefore argue that the United States of 1991 mapped to Great Britain of World War II and that Bush mapped to Winston Churchill, the British Prime Minister (because Bush, similar to Churchill, led his nation and Western allies in early opposition to aggression). These conflicting pressures made the mappings ambiguous. However, the pressure to maintain structural consistency implies that people who mapped the United States to Britain should also tend to map Bush to Churchill, whereas those who mapped the United States to the United States should instead map Bush to Roosevelt.

During the first 2 days of the U.S.-led counterattack against the Iraqi invasion of Kuwait, Spellman and Holyoak (1992) asked a group of American undergraduates a few questions to find out how they interpreted the analogy between the then-current situation in the Persian Gulf and World War II. The undergraduates were asked to suppose that Saddam Hussein was analogous to Hitler. Regardless of whether they believed the analogy was appropriate, they were then asked to write down the most natural match in the World War II situation for Iraq, the United States, Kuwait, Saudi Arabia, and George Bush. For those students who gave evidence that they knew the basic facts about World War II, the majority produced mappings that fell into one of two patterns. Those students who mapped the United States to itself also mapped Bush to Roosevelt; these same students also tended to map Saudi Arabia to Great Britain. Other students, in contrast, mapped the United States to Great Britain and Bush to Churchill, which in turn (so as to maintain one-to-one correspondences) forced Saudi Arabia to map to some country other than Britain. The mapping for Kuwait (which did not depend on the choice of mappings for Bush, the United States, or Saudi Arabia) was usually to one or two of the early victims of Germany in World War II (usually Austria or Poland).

The analogy between the Persian Gulf situation and World War II thus generated a "bistable" mapping: People tended to provide mappings based on either of two coherent but mutually incompatible sets of correspondences. Spellman and Holyoak (1992) went on to perform a second study, using a different group of undergraduates, to show that people's preferred mappings could be pushed around by manipulating their knowledge of the source analog, World War II. Because many undergraduates were lacking in knowledge about the major participants and events in World War II, it proved possible to "guide" them to one or the other mapping pattern by having them first read a slightly biased summary of events in World War II. The various summaries were all historically "correct," in the sense of providing only information taken directly from history books, but each contained slightly different information and emphasized different points. Each summary began with an identical passage about Hitler's acquisition of Austria, Czechoslovakia, and Poland and the efforts by Britain and France to stop him. The versions then diverged. Some versions went on to emphasize the personal role of Churchill and the national role of Britain; other versions placed greater emphasis on what Roosevelt and the United States did to further the war effort. After reading one of these summaries of World War II, the undergraduates were asked the same mapping questions as had been used in the previous study. The same bistable mapping patterns emerged as before, but this time the summaries influenced which of the two coherent patterns of responses students tended to give. People who read a "Churchill" version tended to map Bush to Churchill and the United States to Great Britain, whereas those who read a "Roosevelt" version tended to map Bush to Roosevelt and the United States to the United States. It thus appears that even when an analogy is messy and ambiguous, the constraints on analogical coherence produce predictable interpretations of how the source and target fit together.

Achieving analogical coherence in mapping does not, of course, guarantee that the source will provide a clear and compelling basis for planning a course of action to deal with the target situation. In 1991, President Bush considered Hussein enough of a Hitler to justify intervention in Kuwait but not enough of one to warrant his removal from power in Iraq. A decade later his son, President George W. Bush, reinvoked the World War II analogy to justify a preemptive invasion of Iraq itself. Bush claimed (falsely, as was later revealed) that Hussein was acquiring biological and perhaps nuclear weapons that posed an imminent threat to the United States and its allies. Historical analogies can be used to obfuscate as well as to illuminate.

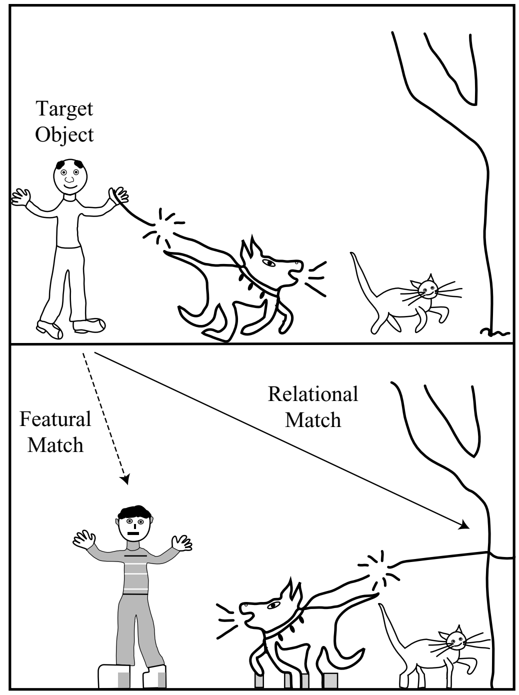

Figure 6.3. An example of a pair of pictures used in studies of analogical mapping with arrows added to indicate featural and relational responses. (From Tohill & Holyoak, 2000, p. 31. Reprinted by permission.)

WORKING MEMORY IN ANALOGICAL MAPPING

Analogical reasoning, because it depends on manipulating structured representations of knowledge, would be expected to make critical use of working memory. The role of working memory in analogy has been explored using a picture-mapping paradigm introduced by Markman and Gentner (1993). An example of stimuli similar to those they used is shown in Figure 6.3. In their experiments, college students were asked to examine the two pictures and then decide (for this hypothetical example) what object in the bottom picture best goes with the man in the top picture. When this single mapping is considered in isolation, people often indicate that the boy in the bottom picture goes with the man in the top picture based on perceptual and semantic similarity of these elements. However, when people are asked to match not just one object but three (e.g., the man, dog, and the tree in the top picture to objects in the bottom picture), they are led to build an integrated representation of the relations among the objects and of higher-order relations between relations. In the top picture, a man is unsuccessfully trying to restrain a dog, which then chases the cat. In the bottom picture, the tree is unsuccessful in restraining the dog, which then chases the boy. Based on these multiple interacting relations, the preferred match to the man in the top picture is not the boy in the lower scene but the tree. Consequently, people who map three objects at once are more likely to map the man to the tree on the basis of their similar relational roles than are people who map the man alone.

Whereas Markman and Gentner (1993) showed that the number of objects to be mapped influences the balance between the impact of element similarity versus relational structure, other studies using the picture-mapping paradigm have demonstrated that manipulations that constrict working memory resources have a similar impact. Waltz, Lau, Grewal, and Holyoak (2000) asked college students to map pictures while performing a secondary task designed to tax working memory (e.g., generating random digits). Adding a dual task diminished relational responses and increased similarity-based responses (see Morrison, Chap. 19). A manipulation that increases people's anxiety level (performing mathematical calculations under speed pressure prior to the mapping task) yielded a similar shift in mapping responses (Tohill & Holyoak, 2000). Most dramatically, degeneration of the frontal lobes radically impairs relation-based mapping (Morrison et al., 2004). In related work using complex story analogs, Krawczyk, Holyoak, and Hummel (2004) demonstrated that mappings (and inferences) based on element similarity versus relational structure were made about equally often when the element similarities were salient and the relational structure was highly complex. All these findings support the hypothesis that mapping on the basis of relations requires adequate working memory to represent and manipulate role bindings (Hummel & Holyoak, 1997).

Inference and Relational Generalization

COPY WITH SUBSTITUTION AND GENERATION

Analogical inference – using a source analog to form a new conjecture, whether it be a step toward solving a math problem (Reed, Dempster, & Ettinger, 1985; see Novick & Bassok, Chap. 14), a scientific hypothesis (see Dunbar & Fugelsang, Chap. 29), a diagnosis for puzzling medical symptoms (see Patel, Arocha, & Zhang, Chap. 30), or a basis for deciding a legal case (see Ellsworth, Chap. 28) – is the fundamental purpose of analogical reasoning. Mapping serves to highlight correspondences between the source and target, including "alignable differences" (Markman & Gentner, 1993) – the distinct but corresponding elements of the two analogs. These correspondences provide the input to an inference engine that generates new target propositions. The basic form of analogical inference has been called "copy with substitution and generation" (CWSG; Holyoak et al., 1994). CWSG involves constructing target analogs of unmapped source propositions by substituting the corresponding target element, if known, for each source element, and if no corresponding target element exists, postulating one as needed. This procedure gives rise to two important corollaries concerning inference errors. First, if critical elements are difficult to map (e.g., because of strong representational asymmetries such as those that hinder mapping a discrete set of elements to a continuous variable; Bassok & Holyoak, 1989; Bassok & Olseth, 1995), then no inferences can be constructed. Second, if elements are mismapped, predictable inference errors will result (Holyoak et al., 1994; Reed, 1987).

All major computational models of analogical inference use some variant of CWSG (e.g., Falkenhainer et al., 1989; Halford et al., 1994; Hofstadter & Mitchell, 1994; Holyoak et al., 1994; Hummel & Holyoak, 2003; Keane & Brayshaw, 1988; Kokinov & Petrov, 2001). CWSG is critically dependent on variable binding and mapping; hence, models that lack these key computational properties (e.g., traditional connectionist models) fail to capture even the most basic aspects of analogical inference (see Doumas & Hummel, Chap. 4).

Athough all analogy models use some form of CWSG, additional constraints on this inference mechanism are critical (Clement & Gentner, 1991; Holyoak et al., 1994; Markman, 1997). If CWSG were unconstrained, then *any* unmapped source proposition would generate an inference about the target. Such a loose criterion for inference generation would lead to rampant errors whenever the source was not isomorphic to a subset of the target, and such isomorphism will virtually never hold for problems of realistic complexity. Several constraints on CWSG were demonstrated in a study by Lassaline (1996; also see Clement & Gentner, 1991; Spellman & Holyoak, 1996). Lassaline had college students read analogs describing properties of hypothetical animals and then rate various possible target inferences for the probability that the conclusion would be true given the information in the premise. Participants rated potential inferences as more probable when the source and target analogs shared more attributes, and hence, mapped more strongly. In addition, their ratings were sensitive to structural and pragmatic constraints. The presence of a higher-order linking relation in the source made an inference more credible. For example, if the source and target animals were both described as having an acute sense of smell, and the source animal was said to have a weak immune system that "develops before" its acute sense of smell, then the inference that the target animal also has a weak immune system would be bolstered relative to stating only that the source animal had an acute sense of smell "and" a weak immune system. The benefit conveyed by the higher-order relation was increased if the relation was explicitly causal (e.g., in the source animal, a weak immune system "causes" its acute sense of smell), rather than less clearly causal ("develops before"). (See Hummel & Holyoak, 2003, for a simulation of this and other inference results using a CWSG algorithm.)

An important question is when analogical inferences are made and how inferences generated by CWSG relate to facts about the target analog that are stated directly. One extreme possibility is that people only make analogical inferences when instructed to do so and that inferences are carefully "marked" as such so they will never be confused with known facts about the target. At the other extreme, it is possible that some analogical inferences are triggered when the target is first processed (given that the source has been activated) and that such inferences are then integrated with prior knowledge of the target. One paradigm for addressing this issue is based on testing for false "recognition" of potential inferences in a subsequent memory test. The logic of the recognition paradigm (Bransford, Barclay, & Franks, 1972) is that if an inference has been made and integrated with the rest of the target analog, then later the reasoner will falsely believe that the inference had been directly presented.

Early work by Schustack and Anderson (1979) provided evidence that people sometimes falsely report that analogical inferences were actually presented as facts. Blanchette and Dunbar (2002) performed a series of experiments designed to assess when analogical inferences are made. They had college students (in Canada) read a text describing a current political issue, possible legalization of marijuana use, which served as the target analog. Immediately afterward, half the students read, "The situation with marijuana can be compared to . . . ", followed by an additional text describing the period early in the twentieth century when alcohol use was prohibited. Importantly, the students in the analogy condition were not told how prohibition mapped onto the marijuana debate, nor were they asked to draw any inferences. After a delay (1 week in one experiment, 15 minutes in another), the students were given a list of sentences and were asked to decide whether each sentence had actually been presented in the text about marijuana use. The critical items were sentences such as "The government could set up agencies to control the quality and take over the distribution of marijuana." These sentences had never been presented; however, they could be generated as analogical inferences by CWSG based on a parallel statement contained in the source analog ("The government set up agencies to control the quality and take over the distribution of alcohol"). Blanchette and Dunbar found that students in the analogy condition said "yes" to analogical inferences about 50% of the time, whereas control subjects who had not read the source analog about prohibition said "yes" only about 25% of the time. This tendency to falsely "recognize" analogical inferences that had never been read was obtained both after long and short delays and with both familiar and less familiar materials.

It thus appears that when people notice the connection between a source and target, and they are sufficiently engaged in an effort to understand the target situation, analogical inferences will be generated by CWSG and then integrated with prior knowledge of the target. At least sometimes, an analogical inference becomes accepted as a stated fact. This result obviously has important implications for understanding analogical reasoning, such as its potential for use as a tool for persuasion.

RELATIONAL GENERALIZATION

In addition to generating local inferences about the target by CWSG, analogical reasoning can give rise to relational generalizations – abstract schemas that establish an explicit representation of the commonalities between the source and the target. Comparison of multiple analogs can result in the induction of a schema, which in turn will facilitate subsequent transfer to additional analogs. The induction of such schemas has been demonstrated in both adults (Catrambone & Holyoak, 1989; Gick & Holyoak, 1983; Loewenstein, Thompson, & Gentner, 1999; Ross & Kennedy, 1990) and young children (Brown, Kane, & Echols, 1986; Chen & Daehler, 1989; Holyoak et al., 1984; Kotovsky & Gentner, 1996). People are able to induce schemas by comparing just two analogs to one another (Gick & Holyoak, 1983). Indeed, people will form schemas simply as a side effect of applying one solved source problem to an unsolved target problem (Novick & Holyoak, 1991; Ross & Kennedy, 1990).

In the case of problem schemas, more effective schemas are formed when the goal-relevant relations are the focus rather than incidental details (Brown et al., 1986; Brown, Kane, & Long, 1989; Gick & Holyoak, 1983). In general, any kind of processing that helps people focus on the underlying causal structure of the analogs, thereby encouraging learning of more effective problem schemas, will improve subsequent transfer to new problems. For example, Gick and Holyoak (1983) found that induction of a "convergence" schema from two disparate analogs was facilitated when each story stated the underlying solution principle abstractly: "If you need a large force to accomplish some purpose, but are prevented from applying such a force directly, many smaller forces applied simultaneously from different directions may work just as well." In some circumstances, transfer can also be improved by having the reasoner generate a problem analogous to an initial example (Bernardo, 2001). Other work has shown that abstract diagrams that highlight the basic idea of using multiple converging forces can aid in schema induction and subsequent transfer (Beveridge & Parkins, 1987; Gick & Holyoak, 1983) – especially when the diagram uses motion cues to convey perception of forces acting on a central target (Pedone, Hummel, & Holyoak, 2001; see Figure 6.4, top).

Although two examples can suffice to establish a useful schema, people are able to incrementally develop increasingly abstract schemas as additional examples are provided (Brown et al., 1986, 1989; Catrambone & Holyoak, 1989). However, even with multiple examples that allow novices to start forming schemas, people may still fail to transfer the analogous solution to a problem drawn from a different domain if a substantial delay intervenes or if the context is changed (Spencer & Weisberg, 1986). Nonetheless, as novices continue to develop

P1: JZZ
0521824176c06.xml CB798B/Holyoak 0 521 82417 6 December 26, 2004 12:49

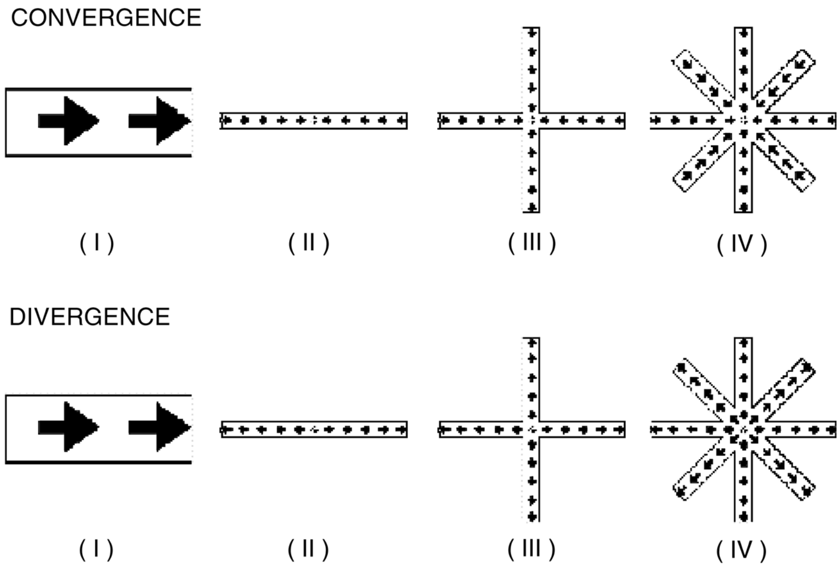

Figure 6.4. Sequence of diagrams used to convey the convergence schema by perceived motion. Top: sequence illustrating convergence (arrows appear to move inward in II–IV). Bottom: control sequence in which arrows diverge instead of converge (arrows appear to move outward in II–IV). (From Pedone, Holyoak, & Hummel, 2001, p. 217. Reprinted by permission.)

more powerful schemas, long-term transfer in an altered context can be dramatically improved (Barnett & Koslowski, 2002). For example, Catrambone and Holyoak (1989) gave college students a total of three convergence analogs to study, compare, and solve. The students were first asked a series of detailed questions designed to encourage them to focus on the abstract structure common to two of the analogs. After this abstraction training, the students were asked to solve another analog from a third domain (not the tumor problem), after which they were told the convergence solution to it (which most students were able to generate themselves). Finally, 1 week later, the students returned to participate in a different experiment. After the other experiment was completed, they were given the tumor problem to solve. More than 80% of participants came up with the converging rays solution without any hint. As the novice becomes an expert, the emerging schema becomes increasingly accessible and is triggered by novel problems that share its structure. Deeper similarities have been constructed between analogous situations that fit the schema. As schemas are acquired from examples, they in turn guide future mappings and inferences (Bassok, Wu, & Olseth, 1995).

Computational Models of Analogy

From its inception, work on analogy in relation to knowledge representation has involved the development of detailed computational models of the various components of analogical reasoning typically focusing on the central process of structure mapping. The most influential early models included SME (Structure Mapping Engine; Falkenhainer, Forbus, & Gentner, 1989), ACME (Analogical Mapping by Constraint Satisfaction; Holyoak & Thagard, 1989a), IAM (Incremental Analogy Model; Keane & Brayshaw, 1988), and Copycat (Hofstadter & Mitchell, 1994). More recently, models of analogy have been developed based on knowledge representations constrained by neural mechanisms (Hummel & Holyoak, 1992). These efforts included an approach based on the use of tensor products for variable binding, the STAR model (Structured Tensor Analogical Reasoning; Halford et al., 1994; see Halford, Chap. 22), and another based on neural synchrony, the LISA model (Learning and Inference with Schemas and Analogies; Hummel & Holyoak, 1997, 2003; see Doumas & Hummel, Chap. 4). (For a brief overview of computational models of analogy, see French, 2002.) Three models are sketched to illustrate the general nature of computational approaches to analogy.

Structure Mapping Engine (SME)

SME (Falkenhainer et al., 1989) illustrates how analogical mapping can be performed by algorithms based on partial graph matching. The basic knowledge representation for the inputs is based on a notation in the style of predicate calculus. If one takes a simple example based on the World War II analogy as it was used by President George Bush in 1991, a fragment might look like

SOURCE:
Führer-of (Hitler, Germany)
occupy (Germany, Austria)
evil (Hitler)
cause [evil (Hitler), occupy (Germany, Austria)]
prime-minister-of (Churchill, Great Britain)
cause [occupy (Germany, Austria), counterattack (Churchill, Hitler)]

TARGET:
president-of (Hussein, Iraq)
invade (Iraq, Kuwait)
evil (Hussein)
cause [evil (Hussein), invade (Iraq, Kuwait)]
president-of (Bush, United States)

SME distinguishes objects (role fillers, such as "Hitler"), attributes (one-place predicates, such as "evil" with its single role filler), first-order relations (multiplace predicates, such as "occupy" with its two role fillers), and higher-order relations (those such as "cause" that take at least one first-order relation as a role filler). As illustrated in Figure 6.5, the predicate-calculus notation is equivalent to a graph structure. An analogical mapping can then be viewed as a set of correspondences between partially matching graph structures.

The heart of the SME algorithm is a procedure for finding graph matches that satisfy certain criteria. The algorithm operates in three stages, progressing in a "local-to-global" direction. First, SME proposes local matches between all identical predicates and their associated role fillers. It is assumed similar predicates (e.g., "Führer-of" and "president-of"; "occupy" and "invade") are first transformed into more general predicates (e.g.,"leader-of"; "attack") that reveal a hidden identity. (In practice, the programmer must make the required substitutions so similar but nonidentical predicates can be matched.) The resulting matches are typically inconsistent in that one element in the source may match multiple elements in the target (e.g., Hitler might match either Hussein or Bush because all are "leaders"). Second, the resulting local matches are integrated into structurally consistent clusters or "kernels" (e.g., the possible match between Hitler and Bush is consistent with that between Germany and the United States, and so these matches would form part of a single kernel). Third, the kernels are merged into a small number of sets that are maximal in size (i.e., that include matches between the greatest number of nodes in the two graphs), while maintaining correspondences that are structurally consistent and one to one. SME then ranks the resulting sets of mappings by a structural evaluation metric that favors "deep" mappings (ones that include correspondences between higher-order relations). For our example, the optimal set will respectively map Hitler, Germany, Churchill, and Great Britain to Hussein, Iraq, Bush, and the United States because of the support provided by the mapping between the higher-order "cause" relations involving "occupy/invade." Using this optimal mapping, SME applies a CWSG algorithm to generate inferences about the target based on unmapped propositions in the source. Here, the final "cause" relation

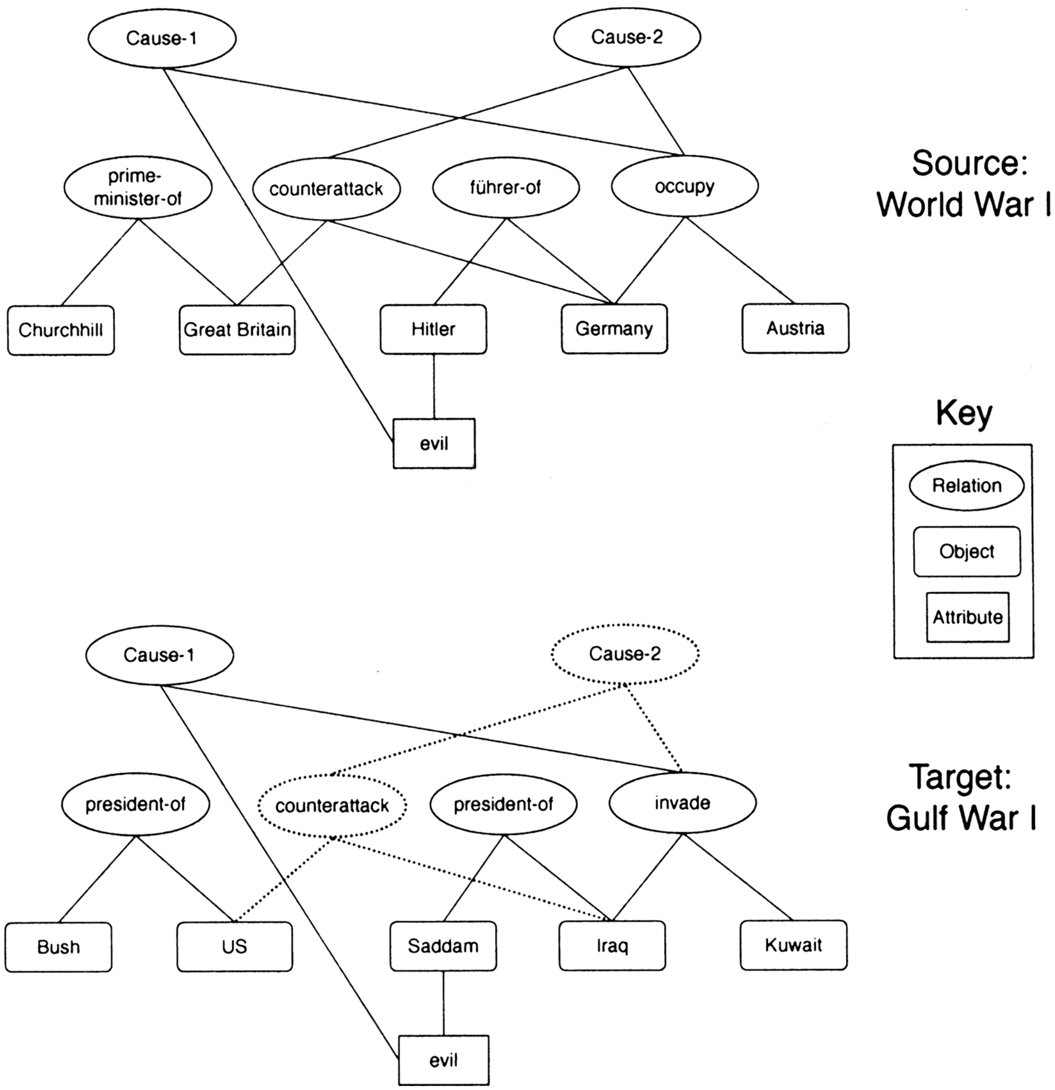

Figure 6.5. SME's graphical representation of a source and target analog.

in the source will yield the analogical inference, *cause [attack (Iraq, Kuwait), counterattack (Bush, Hussein)]*.

SME thus models the mapping and inference components of analogical reasoning. A companion model, MACFAC ("Many Are Called but Few Are Chosen"; Forbus, Gentner, & Law, 1995) deals with the initial retrieval of a source analog from long-term memory. MACFAC has an initial stage ("many are called") in which analogs are represented by *content vectors*, which code the relative number of occurrences of a particular predicate in the corresponding structured representation. (Content vectors are computed automatically from the underlying structural representations.) The content vector for the target is then matched to vectors for all analogs stored in memory, and the dot product for each analog pair is calculated as an index of similarity. The source analog with the highest dot product, plus other stored analogs with relatively high dot products, are marked as retrieved. In its second stage, MACFAC uses SME to assess the degree of the structural overlap between the target and each possible source, allowing the program to identify a smaller number of potential sources that have the highest degrees of structural parallelism with the target ("few are chosen"). As the content vectors used in the first stage of MACFAC do not code role bindings, the model provides a qualitative account of why the retrieval stage of analogy is less sensitive to structure than is the mapping stage.

Analogical Mapping by Constraint Satisfaction (ACME)

The ACME model (Holyoak, Novick, & Melz, 1994; Holyoak & Thagard, 1989a) was directly influenced by connectionist models based on parallel constraint satisfaction (Rumelhart, Smolensky, McClelland, & Hinton, 1986; see Doumas & Hummel, Chap. 4). ACME takes as input symbolic representations of the source and target analogs in essentially the same form as those used in SME. However, whereas SME focuses on structural constraints, ACME instantiates a multiconstraint theory in which structural, semantic, and pragmatic constraints interact to determine the optimal mapping. ACME accepts a numeric code for degree of similarity between predicates, which it uses as a constraint on mapping. Thus, ACME, unlike SME, can match similar predicates (e.g., "occupy" and "invade") without explicitly recoding them as identical. In addition, ACME accepts a numeric code for the pragmatic importance of a possible mapping, which is also used as a constraint.

ACME is based on a constraint satisfaction algorithm, which proceeds in three steps. First, a connectionist "mapping network" is constructed in which the units represent hypotheses about possible element mappings and the links represent specific instantiations of the general constraints (Figure 6.6). Second, an interactive-activation algorithm operates to "settle" the mapping network in order to identify the set of correspondences that collectively represent the "optimal" mapping between the analogs. Any constraint may be locally violated to establish optimal global coherence. Third, if the model is being used to generate inferences and correspondences, CWSG is applied to generate inferences based on the correspondences identified in the second step.

ACME has a companion model, ARCS (Analog Retrieval by Constraint Satisfaction; Thagard, Holyoak, Nelson, & Gochfeld, 1990) that models analog retrieval. Analogs in long-term memory are connected within a semantic network (see Medin & Rips, Chap. 3); this network of concepts provides the initial basis by which a target analog activates potential source analogs. Those analogs in memory that are identified as having semantic links to the target (i.e., those that share similar concepts) then participate in an ACME-like constraint satisfaction process to select the optimal source. The constraint network formed by ARCS is restricted to those concepts in each analog that have semantic links; hence, ARCS shows less sensitivity to structure in retrieval than does ACME in mapping. Because constraint satisfaction algorithms are inherently competitive, ARCS can model the finding that analogical access is more sensitive to structure when similar source analogs in long-term memory compete to be retrieved (Wharton et al., 1994, 1996).

Learning and Inference with Schemas and Analogies (LISA)

Similar to ACME, the LISA model (Hummel & Holyoak, 1997, 2003) is based on the principles of the multiconstraint theory of analogy; unlike ACME, LISA operates within psychologically and neurally realistic constraints on working memory (see Doumas & Hummel, Chap. 4; Morrison, Chap. 19). The models discussed previously include at most localist representations of the meaning of concepts (e.g., a semantic network in the case of ARCS), and most of their processing is performed on propositional representations unaccompanied by any more detailed level of conceptual representation (e.g., neither

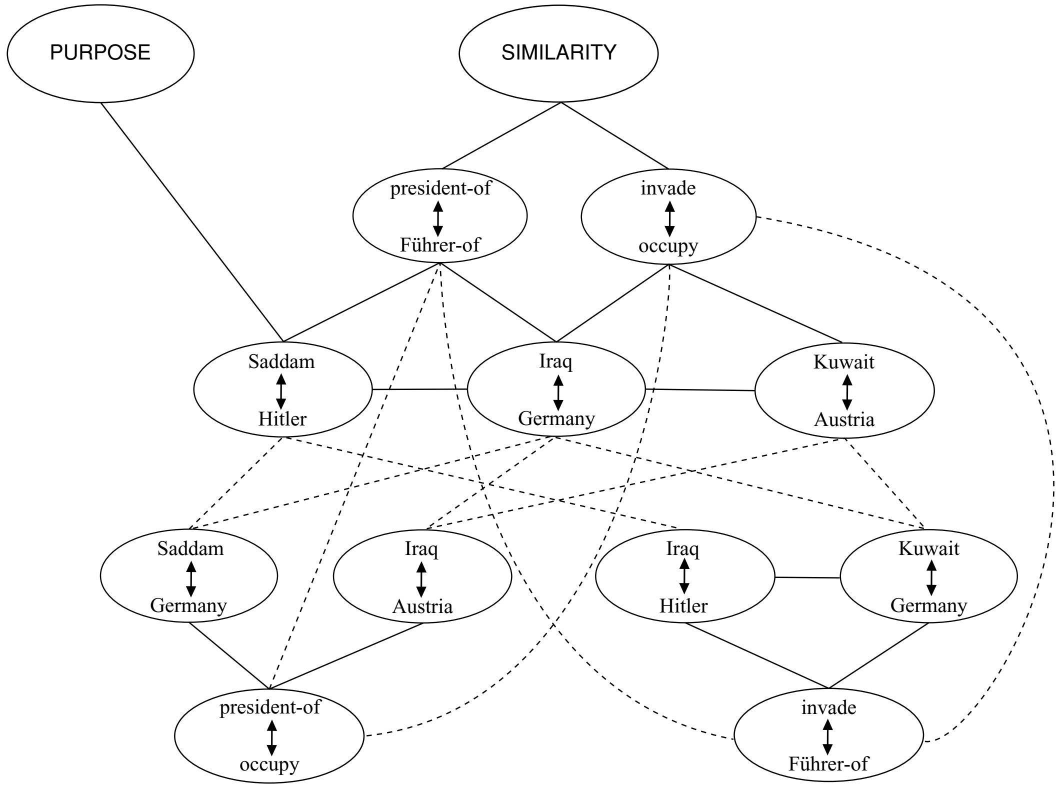

Figure 6.6. A constraint-satisfaction network in ACME.

ACME nor SME includes any representation of the meaning of concepts). LISA also goes beyond previous models in that it provides a unified account of all the major components of analogical reasoning (retrieval, mapping, inference, and relational generalization).

LISA represents propositions using a hierarchy of distributed and localist units (see Figure 4.1 in Doumas & Hummel, Chap. 4). LISA includes both a long-term memory for propositions and concept meanings and a limited-capacity working memory. LISA's working memory representation, which uses neural synchrony to encode role-filler bindings, provides a natural account of the capacity limits of working memory because it is only possible to have a finite number of bindings simultaneously active and mutually *out* of synchrony.

Analog retrieval is accomplished as a form of guided pattern matching. Propositions in a target analog generate synchronized patterns of activation on the semantic units, which in turn activate propositions in potential source analogs residing in long-term memory. The resulting coactivity of source and target elements, augmented with a capacity to learn which structures in the target were coactive with which in the source, serves as the basis for analogical mapping. LISA includes a set of *mapping connections* between units of the same type (e.g., object, predicate) in separate analogs. These connections grow whenever the corresponding units are active simultaneously and thereby permit LISA to learn the correspondences between structures in separate analogs. They also permit correspondences learned early in mapping to influence the correspondences learned later. Augmented with a simple algorithm for self-supervised learning, the mapping algorithm serves as the basis for analogical inference by CWSG. Finally, augmented with a simple algorithm for intersection discovery, self-supervised relational learning serves as the basis for schema induction. LISA has been used to simulate a wide range of data on analogical reasoning (Hummel & Holyoak, 1997, 2003), including both behavioral and neuropsychological studies (Morrison et al., 2004).

Conclusions and Future Directions

When we think analogically, we do much more than just compare two analogs based on obvious similarities between their elements. Rather, analogical reasoning is a complex process of retrieving structured knowledge from long-term memory, representing and manipulating role-filler bindings in working memory, performing self-supervised learning to form new inferences, and finding structured intersections between analogs to form new abstract schemas. The entire process is governed by the core constraints provided by isomorphism, similarity of elements, and the goals of the reasoner (Holyoak & Thagard, 1989a). These constraints apply in all components of analogical reasoning: retrieval, mapping, inference, and relational generalization. When analogs are retrieved from memory, the constraint of element similarity plays a large role, but relational structure is also important – especially when multiple source analogs similar to the target are competing to be selected. For mapping, structure is the most important constraint but requires adequate working memory resources; similarity and purpose also contribute. The success of analogical inference ultimately depends on whether the purpose of the analogy is achieved, but satisfying this constraint is intimately connected with the structural relations between the analogs. Finally, relational generalization occurs when schemas are formed from the source and target to capture those structural patterns in the analogs that are most relevant to the reasoner's purpose in exploiting the analogy.

Several current research directions are likely to continue to develop. Computational models of analogy, such as LISA (Hummel & Holyoak, 1997, 2003), have begun to connect behavioral work on analogy with research in cognitive neuroscience (Morrison et al., 2004). We already have some knowledge of the general neural circuits that underlie analogy and other forms of reasoning (see Goel, Chap. 20). As more sophisticated noninvasive neuroimaging methodologies are developed, it should become possible to test detailed hypotheses about the neural mechanisms underlying analogy, such as those based on temporal properties of neural systems.

Most research and modeling in the field of analogy has emphasized quasilinguistic knowledge representations, but there is good reason to believe that reasoning in general has close connections to perception (e.g., Pedone et al., 2001). Perception provides an important starting point for grounding at least some "higher" cognitive representations (Barsalou, 1999). Some progress has been made in integrating analogy with perception. For example, the LISA model has been augmented with a Metric Array Module (MAM; Hummel & Holyoak, 2001), which provides specialized processing of metric information at a level of abstraction applicable to both perception and quasispatial concepts. However, models of analogy have generally failed to address evidence that the difficulty of solving problems and transferring solution methods to isomorphic problems is dependent on the difficulty of perceptually encoding key relations. The ease of solving apparently isomorphic problems (e.g., isomorphs of the well-known Tower of Hanoi) can vary enormously, depending on perceptual cues (Kotovsky & Simon, 1990; see Novick & Bassok, Chap. 14).

More generally, models of analogy have not been well integrated with models of problem solving (see Novick & Bassok, Chap. 14), even though analogy clearly affords an important mechanism for solving problems. In its general form, problem solving requires sequencing multiple operators, establishing subgoals, and using combinations of rules to solve related but nonisomorphic problems. These basic requirements are beyond the capabilities of virtually all computational models of analogy (but see Holyoak & Thagard, 1989b, for an early although limited effort to integrate analogy within a rule-based problem-solving system). The most successful models of human problem solving have been formulated as production systems (see Lovett & Anderson, Chap. 17), and Salvucci and Anderson (2001) developed a model of analogy based on the ACT-R production system. However, this model is unable to solve reliably any analogy that requires integration of multiple relations – a class that includes analogies within the grasp of young children (Halford, 1993; Richland et al., 2004; see Halford, Chap. 22). The integration of analogy models with models of general problem solving remains an important research goal.

Perhaps the most serious limitation of current computational models of analogy is that their knowledge representations must be hand-coded by the modeler, whereas human knowledge representations are formed autonomously. Closely related to the challenge of avoiding hand-coding of representations is the need to flexibly rerepresent knowledge to render potential analogies perspicuous. Concepts often have a close conceptual relationship with more complex relational forms (e.g., Jackendoff, 1983). For example, causative verbs such as *lift* (e.g., "John lifted the hammer") have very similar meanings to structures based on an explicit higher-order relation, *cause* (e.g., "John caused the hammer to rise"). In such cases, the causative verb serves as a "chunked" representation of a more elaborate predicate-argument structure. People are able to "see" analogies even when the analogs have very different linguistic forms (e.g., "John lifted the hammer in order to strike the nail" might be mapped onto "The Federal Reserve used an increase in interest rates as a tool in its efforts to drive down inflation"). A deeper understanding of human knowledge representation is a prerequisite for a complete theory of analogical reasoning.

Acknowledgments

Preparation of this chapter was supported by grants R305H030141 from the Institute of Education Sciences and SES-0080375 from the National Science Foundation. Kevin Dunbar and Robert Morrison provided valuable comments on an earlier draft.

References

Barnett, S. M., & Koslowski, B. (2002). Solving novel, ill-defined problems: Effects of type of experience and the level of theoretical understanding it generates. *Thinking & Reasoning*, 8, 237–267.

Barsalou, L. W. (1999). Perceptual symbol systems. *Behavioral and Brain Sciences*, 22, 577–660.

Bassok, M., & Holyoak, K. J. (1989). Interdomain transfer between isomorphic topics in algebra and physics. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 15, 153–166.

Bassok, M., & Olseth, K. L. (1995). Object-based representations: Transfer between cases of continuous and discrete models of change. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 21, 1522–1538.

Bassok, M., Wu, L. L., & Olseth, K. L. (1995). Judging a book by its cover: Interpretative effects of content on problem-solving transfer. *Memory & Cognition*, 23, 354–367.

Bernardo, A. B. I. (2001). Principle explanation and strategic schema abstraction in problem solving. *Memory & Cognition*, 29, 627–633.

Beveridge, M., & Parkins, E. (1987). Visual representation in analogical problem solving. *Memory & Cognition*, 15, 230–237.

Black, M. (1962). *Models and metaphors*. Ithaca, NY: Cornell University Press.

Blanchette, I., & Dunbar, K. (2000). Analogy use in naturalistic settings: The influence of audience, emotion, and goal. *Memory & Cognition*, 29, 730–735.

Blanchette, I., & Dunbar, K. (2001). How analogies are generated: The roles of structural and superficial similarity. *Memory & Cognition*, 28, 108–124.

Blanchette, I., & Dunbar, K. (2002). Representational change and analogy: How analogical inferences alter target representations. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 28, 672–685.

Boroditsky, L. (2000). Metaphoric structuring: Understanding time through spatial metaphors. *Cognition*, 75, 1–28.

Bransford, J. D., Barclay, J. R., & Franks, J. J. (1972). Sentence memory: A constructive versus interpretive approach. *Cognitive Psychology*, 3, 193–209.

Brown, A. L., Kane, M. J., & Echols, C. H. (1986). Young children's mental models determine analogical transfer across problems with a common goal structure. *Cognitive Development*, 1, 103–121.

Brown, A. L., Kane, M. J., & Long, C. (1989). Analogical transfer in young children: Analogies as tools for communication and exposition. *Applied Cognitive Psychology*, 3, 275–293.

Bunge, S. A., Wendelken, C., Badre, D., & Wagner, A. D. (2004). Analogical reasoning and prefrontal cortex: Evidence for separable retrieval and integration mechanisms. *Cerebral Cortex*, *x*.

Carbonell, J. G. (1983). Learning by analogy: Formulating and generalizing plans from past experience. In R. S. Michalski, J. G. Carbonell, & T. M. Mitchell (Eds.), *Machine learning: An artificial intelligence approach* (pp. 137–161). Palo Alto, CA: Tioga.

Carpenter, P. A., Just, M. A., & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Progressive Matrices test. *Psychological Review*, 97, 404–431.

Catrambone, R., & Holyoak, K. J. (1989). Overcoming contextual limitations on problem-solving transfer. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 15, 1147–1156.

Cattell, R. B. (1971). *Abilities: Their structure, growth, and action*. Boston, MA: Houghton-Mifflin.

Chen, Z. (1996). Children's analogical problem solving: The effects of superficial, structural, and procedural similarity. *Journal of Experimental Child Psychology*, 62, 410–431.

Chen, Z., & Daehler, M. W. (1989). Positive and negative transfer in analogical problem solving by 6-year-old children. *Cognitive Development*, 4, 327–344.

Clement, C. A., & Gentner, D. (1991). Systematicity as a selection constraint in analogical mapping. *Cognitive Science*, 15, 89–132.

Duncan, J., Seitz, R. J., Kolodny, J., Bor, D., Herzog, H., Ahmed, A., Newell, F. N., & Emslie, H. (2000). A neural basis for general intelligence. *Science*, 289, 457–460.

Duncker, K. (1945). On problem solving. *Psychological Monographs*, 58 (Whole No. 270).

Evans, T. G. (1968). A program for the solution of geometric-analogy intelligence test questions. In M. Minsky (Ed.), *Semantic information processing* (pp. 271–353). Cambridge, MA: MIT Press.

Falkenhainer, B., Forbus, K. D., & Gentner, D. (1989). The structure-mapping engine: Algorithm and examples. *Artificial Intelligence*, 41, 1–63.

Fauconnier, G. (2001). Conceptual blending and analogy. In D. Gentner, K. J. Holyoak, & B. N. Kokinov (Eds.), *The analogical mind: Perspectives from cognitive science* (pp. 255–285). Cambridge, MA: MIT Press.

Fauconnier, G., & Turner, M. (1998). Conceptual integration networks. *Cognitive Science*, 22, 133–187.

Forbus, K. D., Gentner, D., & Law, K. (1995). MAC/FAC: A model of similarity-based retrieval. *Cognitive Science*, 19, 141–205.

French, R. M. (2002). The computational modeling of analogy-making. *Trends in Cognitive Science*, 6, 200–205.

Gentner, D. (1982). Are scientific analogies metaphors? In D. S. Miall (Eds.), *Metaphor: Problems and perspectives* (pp. 106–132). Brighton, UK: Harvester Press.

Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science*, 7, 155–170.

Gentner, D., Bowdle, B., Wolff, P., & Boronat, C. (2001). Metaphor is like analogy. In D. Gentner, K. J. Holyoak, & B. N. Kokinov (Eds.), *The analogical mind: Perspectives from cognitive science* (pp. 199–253). Cambridge, MA: MIT Press.

Gentner, D., Falkenhainer, B., & Skorstad, J. (1988). Viewing metaphor as analogy. In D. H. Helman (Eds.), *Analogical reasoning: Perspectives of artificial intelligence, cognitive science, and philosophy* (pp. 171–177). Dordrecht, The Netherlands: Kluwer.

Gentner, D., & Gentner, D. R. (1983). Flowing waters or teeming crowds: Mental models of electricity. In D. Gentner & A. L. Stevens (Eds.), *Mental models* (pp. 99–129). Hillsdale, NJ: Erlbaum.

Gentner, D., & Rattermann, M. (1991). Language and the career of similarity. In S. A. Gelman & J. P. Byrnes (Eds.), *Perspectives on thought and language: Interrelations in development* (pp. 225–277). Cambridge, UK Cambridge University Press.

Gentner, D., Rattermann, M., & Forbus, K. (1993). The roles of similarity in transfer: Separating retrievability from inferential soundness. *Cognitive Psychology*, 25, 524–575.

Gentner, D., & Toupin, C. (1986). Systematicity and surface similarity in the development of analogy. *Cognitive Science*, 10, 277–300.

Gick, M. L., & Holyoak, K. J. (1980). Analogical problem solving. *Cognitive Psychology*, 12, 306–355.

Gick, M. L., & Holyoak, K. J. (1983). Schema induction and analogical transfer. *Cognitive Psychology*, 15, 1–38.

Glucksberg, S., Gildea, P., & Bookin, H. (1982). On understanding nonliteral speech: Can people ignore metaphors? *Journal of Verbal Learning and Verbal Behaviour*, 21, 85–98.

Glucksberg, S., & Keysar, B. (1990). Understanding metaphorical comparisons: Beyond similarity. *Psychological Review*, 97, 3–18.

Glucksberg, S., McClone, M. S., & Manfredi, D. (1997). Property attribution in metaphor comprehension. *Journal of Memory and Language*, 36, 50–67.

Goswami, U. (1992). *Analogical reasoning in children*. Hillsdale, NJ: Erlbaum.

Goswami, U. (2001). Analogical reasoning in children. In D. Gentner, K. J. Holyoak, & B. N. Kokinov (Eds.), *The analogical mind: Perspectives from cognitive science* (pp. 437–470). Cambridge, MA: MIT Press.

Halford, G. S. (1993). *Children's understanding: The development of mental models*. Hillsdale, NJ: Erlbaum.

Halford, G. S., & Wilson, W. H. (1980). A category theory approach to cognitive development. *Cognitive Psychology*, 12, 356–411.

Halford, G. S., Wilson, W. H., Guo, J., Gayler, R. W., Wiles, J., & Stewart, J. E. M. (1994). Connectionist implications for processing capacity limitations in analogies. In K. J. Holyoak & J. A. Barnden (Eds.), *Advances in connectionist and neural computation theory, Vol. 2: Analogical connections* (pp. 363–415). Norwood, NJ: Ablex.

Hesse, M. (1966). *Models and analogies in science*. Notre Dame, IN: Notre Dame University Press.

Hofstadter, D. R., & Mitchell, M. (1994). The Copycat project: A model of mental fluidity and analogy-making. In K. J. Holyoak & J. A. Barnden (Eds.), *Analogical connections. Advances in connectionist and neural computation theory* (Vol. 2, pp. 31–112). Norwood, NJ: Ablex.

Holland, J. H., Holyoak, K. J., Nisbett, R. E., & Thagard, P. (1986). *Induction: Processes of inference, learning, and discovery*. Cambridge, MA: MIT Press.

Holyoak, K. J. (1982). An analogical framework for literary interpretation. *Poetics*, 11, 105–126.

Holyoak, K. J. (1985). The pragmatics of analogical transfer. In G. H. Bower (Ed.), *The psychology of learning and motivation* (Vol. 19, pp. 59–87). New York: Academic Press.

Holyoak, K. J., Junn, E. N., & Billman, D. O. (1984). Development of analogical problem-solving skill. *Child Development*, 55, 2042–2055.

Holyoak, K. J., & Koh, K. (1987). Surface and structural similarity in analogical transfer. *Memory & Cognition*, 15, 332–340.

Holyoak, K. J., Novick, L. R., & Melz, E. R. (1994). Component processes in analogical transfer: Mapping, pattern completion, and adaptation. In K. J. Holyoak & J. A. Barnden (Eds.), *Advances in connectionist and neural computation theory, Vol. 2: Analogical connections* (pp. 113–180). Norwood, NJ: Ablex.

Holyoak, K. J., & Thagard, P. (1989a). Analogical mapping by constraint satisfaction. *Cognitive Science*, 13, 295–355.

Holyoak, K. J., & Thagard, P. (1989b). A computational model of analogical problem solving. In S. Vosniadou & A. Ortony (Eds.), *Similarity and analogical reasoning* (pp. 242–266). New York: Cambridge University Press.

Holyoak, K. J., & Thagard, P. (1995). *Mental leaps: Analogy in creative thought*. Cambridge, MA: MIT Press.

Hummel, J. E., & Holyoak, K. J. (1992). Indirect analogical mapping. In *Proceedings of the Forteenth Annual Conference of the Cognitive Science Society* (pp. 516–521). Hillsdale, NJ: Erlbaum.

Hummel, J. E., & Holyoak, K. J. (1997). Distributed representations of structure: A theory of analogical access and mapping. *Psychological Review*, 104, 427–466.

Hummel, J. E., & Holyoak, K. J. (2001). A process model of human transitive inference. In M. L. Gattis (Ed.), *Spatial schemas in abstract thought* (pp. 279–305). Cambridge, MA: MIT Press.

Hummel, J. E., & Holyoak, K. J. (2003). A symbolic-connectionist theory of relational inference and generalization. *Psychological Review*, *110*, 220–263.

Hunt, E. B. (1974). Quote the raven? Nevermore! In L. W. Gregg (Ed.), *Knowledge and cognition* (pp. 129–157). Hillsdale, NJ: Erlbaum.

Jackendoff, R. (1983). *Semantics and cognition*. Cambridge, MA: MIT Press.

Keane, M. T. (1987). On retrieving analogues when solving problems. *Quarterly Journal of Experimental Psychology*, *39A*, 29–41.

Keane, M. T., & Brayshaw, M. (1988). The incremental analogical machine: A computational model of analogy. In D. Sleeman (Ed.), *European working session on learning* (pp. 53–62). London: Pitman.

Keysar, B. (1989). On the functional equivalence of literal and metaphorical interpretations in discourse. *Journal of Memory and Language*, *28*, 375–385.

Khong, Y. F. (1992). *Analogies at war: Korea, Munich, Dien Bien Phu, and the Vietnam decisions of 1965*. Princeton, NJ: Princeton University Press.

Kokinov, B. N., & Petrov, A. A. (2001). Integration of memory and reasoning in analogy-making: The AMBR model. In D. Gentner, K. J. Holyoak, & B. N. Kokinov (Eds.), *The analogical mind: Perspectives from cognitive science* (pp. 59–124). Cambridge, MA: MIT Press.

Kolodner, J. L. (1983). Reconstructive memory: A computer model. *Cognitive Science*, *7*, 281–328.

Kolodner, J. L. (1993). *Case-based reasoning*. San Mateo, CA: Morgan Kaufmann.

Kotovsky, K., & Simon, H. A. (1990). What makes some problems really hard? Explorations in the problem space of difficulty. *Cognitive Psychology*, *22*, 143–183.

Kotovsky, L., & Gentner, D. (1996). Comparison and categorization in the development of relational similarity. *Child Development*, *67*, 2797–2822.

Krawczyk, D. C., Holyoak, K. J., & Hummel, J. E. (2004). Structural constraints and object similarity in analogical mapping and inference. *Thinking & Reasoning*, *10*, 85–104.

Kroger, J. K., Saab, F. W., Fales, C. L., Bookheimer, S. Y., Cohen, M. S., & Holyoak, K. J. (2002). Recruitment of anterior dorsolateral prefrontal cortex in human reasoning: A parametric study of relational complexity. *Cerebral Cortex*, *12*, 477–485.

Lakoff, G., & Johnson, M. (1980). *Metaphors we live by*. Chicago: University of Chicago Press.

Lakoff, G., & Turner, M. (1989). *More than cool reason: A field guide to poetic metaphor*. Chicago: University of Chicago Press.

Lassaline, M. E. (1996). Structural alignment in induction and similarity. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *22*, 754–770.

Loewenstein, J., Thompson, L., & Gentner, D. (1999). Analogical encoding facilitates knowledge transfer in negotiation. *Psychonomic Bulletin and Review*, *6*, 586–597.

Luo, Q., Perry, C., Peng, D., Jin, Z., Xu, D., Ding, G., & Xu, S. (2003). The neural substrate of analogical reasoning: An fMRI study. *Cognitive Brain Research*, *17*, 527–534.

Markman, A. B. (1997). Constraints on analogical inference. *Cognitive Science*, *21*, 373–418.

Markman, A. B., & Gentner, D. (1993). Structural alignment during similarity comparisons. *Cognitive Psychology*, *23*, 431–467.

Minsky, M. (1975). A framework for representing knowledge. In P. Winston (Ed.), *The psychology of computer vision* (pp. 211–281). New York: McGraw-Hill.

Morrison, R. G., Krawczyk, D. C., Holyoak, K. J., Hummel, J. E., Chow, T. W., Miller, B. L., & Knowlton, B. J. (2004). A neurocomputational model of analogical reasoning and its breakdown in frontotemporal lobar degeneration. *Journal of Cognitive Neuroscience*, *16*, 260–271.

Mulholland, T. M., Pellegrino, J. W., & Glaser, R. (1980). Components of geometric analogy solution. *Cognitive Psychology*, *12*, 252–284.

Novick, L. R., & Holyoak, K. J. (1991). Mathematical problem solving by analogy. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *17*, 398–415.

Pask, C. (2003). Mathematics and the science of analogies. *American Journal of Physics*, *71*, 526–534.

Pedone, R., Hummel, J. E., & Holyoak, K. J. (2001). The use of diagrams in analogical problem solving. *Memory & Cognition*, *29*, 214–221.

Prabhakaran, V., Smith, J. A. L., Desmond, J. E., Glover, G. H., & Gabrieli, J. D. E. (1997). Neural substrates of fluid reasoning: An fMRI study of neocortical activation during performance of the Raven's Progressive Matrices Test. *Cognitive Psychology*, 33, 43–63.

Raven, J. C. (1938). *Progressive matrices: A perceptual test of intelligence, individual form*. London: Lewis.

Reed, S. K. (1987). A structure-mapping model for word problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 13, 124–139.

Reed, S. K., Dempster, A., & Ettinger, M. (1985). Usefulness of analogical solutions for solving algebra word problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 11, 106–125.

Reitman, W. (1965). *Cognition and thought*. New York: Wiley.

Richland, L. E., Morrison, R. G., & Holyoak, K. J. (2004). Working memory and inhibition as constraints on children's development of analogical reasoning. In K. Forbus, D. Gentner & T. Regier (Eds.), *Proceedings of the Twenty-sixth Annual Conference of the Cognitive Science Society*. Mahwah, NJ: Erlbaum.

Ross, B. (1987). This is like that: The use of earlier problems and the separation of similarity effects. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 13, 629–639.

Ross, B. (1989). Distinguishing types of superficial similarities: Different effects on the access and use of earlier problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 15, 456–468.

Ross, B. H., & Kennedy, P. T. (1990). Generalizing from the use of earlier examples in problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 16, 42–55.

Rumelhart, D. E., Smolensky, P., McClelland, J. L., & Hinton, G. E. (1986). Schemata and sequential thought processes in PDP models. In J. L. McClelland, D. E. Rumelhart, & the PDP Research Group (Eds.), *Parallel distributed processing: Explorations in the microstructure of cognition* (Vol. 2). Cambridge, MA: MIT Press.

Salvucci, D. D., & Anderson, J. R. (2001). Integrating analogical mapping and general problem solving: The path-mapping theory. *Cognitive Science*, 25, 67–110.

Schank, R. C. (1982). *Dynamic memory*. New York: Cambridge University Press.

Schustack, M. W., & Anderson, J. R. (1979). Effects of analogy to prior knowledge on memory for new information. *Journal of Verbal Learning and Verbal Behavior*, 18, 565–583.

Seifert, C. M., McKoon, G., Abelson, R. P., & Ratcliff, R. (1986). Memory connections between thematically similar episodes. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 12, 220–231.

Snow, R. E., Kyllonen, C. P., & Marshalek, B. (1984). The topography of ability and learning correlations. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (pp. 47–103). Hillsdale, NJ: Erlbaum.

Spearman, C. (1923). *The nature of intelligence and the principles of cognition*. London, UK: Macmillan.

Spearman, C. (1927). *The abilities of man*. New York: Macmillan.

Spearman, C. (1946). Theory of a general factor. *British Journal of Psychology*, 36, 117–131.

Spellman, B. A., & Holyoak, K. J. (1992). If Saddam is Hitler then who is George Bush?: Analogical mapping between systems of social roles. *Journal of Personality and Social Psychology*, 62, 913–933.

Spellman, B. A., & Holyoak, K. J. (1996). Pragmatics in analogical mapping. *Cognitive Psychology*, 31, 307–346.

Spencer, R. M., & Weisberg, R. W. (1986). Context-dependent effects on analogical transfer. *Memory & Cognition*, 14, 442–449.

Sternberg, R. J. (1977). Component processes in analogical reasoning. *Psychological Review*, 84, 353–378.

Thagard, P. (2000). *Coherence in thought and action*. Cambridge, MA: MIT Press.

Thagard, P., Holyoak, K. J., Nelson, G., & Gochfeld, D. (1990). Analog retrieval by constraint satisfaction. *Artificial Intelligence*, 46, 259–310.

Thagard, P., & Shelley, C. (2001). Emotional analogies and analogical inference. In D. Gentner, K. J. Holyoak, & B. N. Kokinov (Eds.), *The analogical mind: Perspectives from cognitive science* (pp. 335–362). Cambridge, MA: MIT Press.

Tohill, J. M., & Holyoak, K. J. (2000). The impact of anxiety on analogical reasoning. *Thinking & Reasoning*, 6, 27–40.

Tunteler, E., & Resing, W. C. M. (2002). Spontaneous analogical transfer in 4-year-olds: A microgenetic study. *Journal of Experimental Child Psychology*, 83, 149–166.

Waltz, J. A., Knowlton, B. J., Holyoak, K. J., Boone, K. B., Mishkin, F. S., de Menezes Santos, M., Thomas, C. R., & Miller, B. L. (1999). A system for relational reasoning in human prefrontal cortex. *Psychological Science*, 10, 119–125.

Waltz, J. A., Lau, A., Grewal, S. K., & Holyoak, K. J. (2000). The role of working memory in analogical mapping. *Memory & Cognition*, 28, 1205–1212.

Wharton, C. M., Grafman, J., Flitman, S. S., Hansen, E. K., Brauner, J., Marks, A., & Honda, M. (2000). Toward neuroanatomical models of analogy: A positron emission tomography study of analogical mapping. *Cognitive Psychology*, 40, 173–197.

Wharton, C. M., Holyoak, K. J., Downing, P. E., Lange, T. E., Wickens, T. D., & Melz, E. R. (1994). Below the surface: Analogical similarity and retrieval competition in reminding. *Cognitive Psychology*, 26, 64–101.

Wharton, C. M., Holyoak, K. J., & Lange, T. E. (1996). Remote analogical reminding. *Memory & Cognition*, 24, 629–643.

Winston, P. H. (1980). Learning and reasoning by analogy. *Communications of the ACM*, 23, 689–703.

Wolff, P., & Gentner, D. (2000). Evidence for role-neutral initial processing of metaphors. *Journal of Experimental Psychology: Leaning, Memory, and Cognition*, 26, 529–541.