

Analogue Problem Solving

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The use of an analogy from a semantically distant domain to guide the problem-solving process was investigated. The representation of analogy in memory and processes involved in the use of analogies were discussed theoretically and explored in five experiments. In Experiment I oral protocols were used to examine the processes involved in solving a problem by analogy. In all experiments subjects who first read a story about a military problem and its solution tended to generate analogous solutions to a medical problem (Duncker's "radiation problem"), provided they were given a hint to use the story to help solve the problem. Transfer frequency was reduced when the problem presented in the military story was substantially disanalogous to the radiation problem, even though the solution illustrated in the story corresponded to an effective radiation solution (Experiment II). Subjects in Experiment III tended to generate analogous solutions to the radiation problem after providing their own solutions to the military problem. Subjects were able to retrieve the story from memory and use it to generate an analogous solution, even when the critical story had been memorized in the context of two distractor stories (Experiment IV). However, when no hint to consider the story was given, frequency of analogous solutions decreased markedly. This decrease in transfer occurred when the story analogy was presented in a recall task along with distractor stories (Experiment IV), when it was presented alone, and when it was presented in between two attempts to solve the problem (Experiment V). Component processes and strategic variations in analogue problem solving were discussed. Issues related to noticing analogies and accessing them in memory were also examined, as was the relationship of analogue reasoning to other cognitive tasks.

INTRODUCTION

Where do new ideas come from? What psychological mechanisms underlie creative insight? This fundamental issue in the study of thought has received a great deal of informal discussion, but little empirical psychological investigation. The anecdotal reports of creative scientists and mathematicians suggest that the development of a new theory frequently depends on noticing and applying an analogy drawn from a different domain of knowledge (Hadamard, 1954). The hydraulic model of the blood circulation system, the planetary model of atomic structure, and the

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"billiard ball" model of gases all represent major scientific theories founded on analogies (Boden, 1977, Chap. 11).

As these examples suggest, fruitful analogies may be based on a mapping of relations between two very disparate domains. In the brainstorming technique of "Synectics," problem-solving groups are trained to actively search for analogies in areas other than that of the target problem (Gordon, 1961). It seems clear, however, that the "semantic distance" between analogous domains can vary a great deal. For example, Polya (1957) suggests that a useful strategy for solving geometry problems is to search for analogous problems *within* the domain of geometry. Analogies drawn within a domain may also play a role in categorization tasks. Classification models based on comparisons of instances, which have been developed both in psychology (Brooks, 1978; Medin & Schaffer, 1978) and in artificial intelligence (Winston, 1975), involve comparisons between objects within a single domain (e.g., geometric patterns). Collins and his associates (Collins, Warnock, Aiello, & Miller, 1975) have used protocol analyses to investigate the role of analogical reasoning by students who have incomplete knowledge of a problem domain such as geography. For example, a student might evaluate whether the region around Santiago, Chile is likely to produce wine by comparing it to a known wine-producing area, such as Northern California, on the relevant geographic dimensions (e.g., latitude, proximity to ocean, type of terrain).

While the process of solving analogy test items of the form A:B::C:D has been studied quite extensively (Sternberg, 1977a, 1977b), there has been little experimental investigation of analogical thinking in more complex problem-solving tasks. Some studies have examined transfer between homomorphic or isomorphic versions of puzzle problems, such as the "missionaries and cannibals" (Reed, Ernst, & Banerji, 1974) and Tower of Hanoi (Hayes & Simon, 1977) puzzles. These are relatively "well-defined" problems (Reitman, 1964; Simon, 1973), in which the initial conditions, legal operations, and goal state are explicitly specified. In contrast, anecdotal reports of the use of analogies typically involve problems that are much less well defined. The present study was designed to investigate the use of analogies between disparate domains as a guide to finding solutions for an ill-defined problem.

THE RADIATION PROBLEM AND ITS ANALOGIES

Our basic experimental procedure was to provide subjects with a story analogy, describing a problem and its solution, and then to observe how subjects used the analogy in solving a subsequent target problem. The target problem was Duncker's (1945) "radiation problem," which in our experiments was stated as follows.

Suppose you are a doctor faced with a patient who has a malignant

tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach the tumor all at once at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to healthy tissue, but they will not affect the tumor either. What type of procedure might be used to destroy the tumor with the rays, and at the same time avoid destroying the healthy tissue?

There are several reasons why the radiation problem seemed especially suitable for use in a study of analogical problem solving. First, it has all the hallmarks of the kind of "ill-defined" problem for which an analogy from a remote domain might trigger a creative insight. The desired goal state is specified only at a relatively abstract level, and the permissible operations that might be used to achieve the goal are left very open ended. As a consequence, the possible solution proposals vary considerably. This made it possible to test for the use of analogies by attempting to influence the specific solutions that subjects would generate.

In addition, we were able to benefit from Duncker's analyses of the performance of subjects who worked on the problem without receiving an analogy. Duncker identified three broad categories of proposed solutions to the radiation problem: (1) reducing the intensity of the rays as they pass through the healthy tissue; (2) avoiding contact between the rays and healthy tissue; and (3) altering the relative sensitivity to rays of the healthy tissue and the tumor (e.g., by immunizing the healthy tissue or sensitizing the tumor). Our analogies were designed to guide subjects toward specific versions of the first two classes of proposals.

Our general aim in the present study, then, was to explore the process by which subjects use analogies between remote domains to generate problem solutions. Consequently, we wrote a series of stories far removed from the medical domain, each involving a military problem and its solution, which were analogous to the radiation problem. We will introduce the various stories as we proceed; all are presented in the Appendixes.

A FRAMEWORK FOR ANALOGICAL PROBLEM SOLVING

It is important to develop a general conceptual framework within which specific issues concerning the role of analogies in problem solving can be formulated. What is meant by analogy, and how can an analogy be used to generate a problem solution?

In order to make our discussion more concrete we will consider the major story analogy used in the present experiments and the correspond-

ing solution to the radiation problem. In the Attack-Dispersion story¹ a general wishes to capture a fortress located in the center of a country. There are many roads radiating outward from the fortress. All have been mined so that while small groups of men can pass over the roads safely, any large force will detonate the mines. A full-scale direct attack is therefore impossible. The general's solution is to divide his army into small groups, send each group to the head of a different road, and have the groups converge simultaneously on the fortress. The analogous solution to the radiation problem is to simultaneously direct multiple low-intensity rays toward the tumor from different directions. In this way the healthy tissue will be left unharmed, but the effects of the multiple low-intensity rays will summate and destroy the tumor.

At an intuitive level the parallels between the Attack-Dispersion story and the radiation problem are clear. Both situations involve an object that must be overcome, surrounded by objects that must be preserved. The target object in each case occupies a topographically central position in its environment. In each situation the protagonist has available a weapon with an effect proportional to the intensity or amount of the weapon that is used, and so on.

How might people represent these analogical relationships and use them to generate a solution to the target problem? This is not an easy question to answer. First of all, both the story and the problem must be read and understood. In attempting to describe this type of analogical problem solving we thus inherit all the problems associated with text comprehension. In particular, perception of analogy hinges on semantic knowledge and inference procedures. Since no general theory of language understanding is available, we must of necessity gloss over many important issues related to the understanding process. However, recent work on story comprehension (Black & Bower, in press; Kintsch, 1974; Kintsch & Van Dijk, 1978; Rumelhart, 1975; Schank & Abelson, 1977; Thorndyke, 1977) may offer some insights into how our story analogies might be represented in memory. Indeed, there appear to be close ties between the concept of analogy and the concept of "schema," which has been widely applied in discussions of prose comprehension. In essence, both an analogy and a schema consist of an organized system of relations. Consequently, the framework for analogical problem solving presented here will draw its conceptual vocabulary from various schema-based models, as well as from Sternberg's (1977a, 1977b) model of component processes involved in analogical reasoning. We will first consider how

¹ In fact, three versions of this story were used in different experiments (see Appendixes I-III). The versions differed only in minor points of wording of no consequence to the present discussion.

analogy might be represented, and then how this representation could be used to generate a solution to a problem.

The Representation of Analogy

A system of representation for analogy must be able to describe a fundamental property of such relational systems, namely, that analogy may be defined at multiple levels of abstraction. For example, at a relatively low level of abstraction the Attack-Dispersion story and the radiation problem have a variety of corresponding details (e.g., small groups of soldiers correspond to low-intensity rays). At a more abstract level, the story and the problem both involve the goal of overpowering an object located in a region that must be preserved.

The multileveled nature of analogy can perhaps be understood in the context of Kintsch and Van Dijk's (1978) theory of prose representation. They argue that the understanding process may involve the iterative application of a set of inference rules that generate increasingly abstract "macrostructure" representations of a prose passage. These macrostructures essentially correspond to summaries of the passage at various levels of generality. In the case of a problem-oriented story such as the Attack-Dispersion story, an abstract level of macrostructure might state a general solution principle (e.g., to destroy a target when direct application of a large force is harmful to the surrounding area, disperse the attacking forces, and have them converge at the target). The process of extracting a solution principle might thus be viewed as a special case of the process of deriving macrostructures for a body of information. While much remains to be learned about how this process operates, the three specific inference rules proposed by Kintsch and Van Dijk (which they term "deletion," "generalization," and "construction") would seem readily applicable to the type of story analogies we are considering here.

Kintsch and Van Dijk emphasize that control processes are required to select a level of macrostructure analysis consistent with the person's processing goals. Similarly, we assume there is an optimal level of abstraction at which analogical relations may be represented in order to effectively guide the solution process. Indeed, an important empirical issue is to determine what factors influence this optimal level of abstraction.

We will now consider in more detail how an analogy between two relational systems might be represented, assuming an appropriate level of macrostructure has been derived. To pursue our example, Table 1 presents our own summary of the Attack-Dispersion story, as well as a summary of the radiation problem and its dispersion solution. These summaries are intended to reflect the major causal connections within both the story and the problem, and to illustrate the major analogical relations

between them. The sentences in Table 1 are numbered to correspond to an approximate propositional analysis presented in Fig. 1. Propositions from the story and from the radiation problem are matched to indicate analogical relations, and the propositions corresponding to the dispersion solution to the radiation problem (which a person would be required to generate) are italicized in both the table and the figure. Note that some of the propositions included in the story summary (e.g., proposition 11) are inferences, which are not directly stated in the original story (see Appendix I).

The notation in Fig. 1 consists of propositional functions, in which predicates are followed by one or more arguments (enclosed in parentheses). The arguments fill various semantic roles, such as agent, ob-

TABLE 1
A Summary of Attack-Dispersion Story and of Corresponding
Solution to Radiation Problem (See Fig. 1)

Proposition number	
Attack-Dispersion story	
1-2	A fortress was located in the center of the country.
2a	Many roads radiated out from the fortress.
3-4	A general wanted to capture the fortress with his army.
5-7	The general wanted to prevent mines on the roads from destroying his army and neighboring villages.
8	As a result the entire army could not attack the fortress along one road.
9-10	However, the entire army was needed to capture the fortress.
11	So an attack by one small group would not succeed.
12	The general therefore divided his army into several small groups.
13	He positioned the small groups at the heads of different roads.
14-15	The small groups simultaneously converged on the fortress.
16	In this way the army captured the fortress.
Radiation problem and dispersion solution ^a	
1'-2'	A tumor was located in the interior of a patient's body.
3'-4'	A doctor wanted to destroy the tumor with rays.
5'-7'	The doctor wanted to prevent the rays from destroying healthy tissue.
8'	As a result the high-intensity rays could not be applied to the tumor along one path.
9'-10'	However, high-intensity rays were needed to destroy the tumor.
11'	So applying one low-intensity ray would not succeed.
12'	<i>The doctor therefore divided the rays into several low-intensity rays.</i>
13'	<i>He positioned the low-intensity rays at multiple locations around the patient's body.</i>
14'-15'	<i>The low-intensity rays simultaneously converged on the tumor.</i>
16'	<i>In this way the rays destroyed the tumor.</i>

^a Italicized propositions summarize the target dispersion solution.

ject, and location. Propositions may themselves serve as arguments, so in several cases one proposition is embedded in another. We make no claims about the logical adequacy or completeness of the representation in Fig. 1; indeed, many of the indicated arguments (e.g., "low-intensity rays") clearly could be further decomposed. However, separation of relations (predicates) and arguments serves to highlight the critical properties of analogy between remote domains: similarity of corresponding relations despite dissimilarity of corresponding arguments.

The notation in Fig. 1 has been augmented by labeled arcs that represent the major causal connections within the Attack-Dispersion story and the radiation problem. The labels are inspired by but not identical to the analysis of causal types offered by Schank and Abelson (1977) (see also Black & Bower, in press). Roughly, a goal can be a "reason" for an action; a state or action can "enable" a subsequent action; an action can "result" in a subsequent state; and a state or action can "prevent" a subsequent action. Again, the adequacy of this analysis is not really critical for our present purpose, which is simply to make salient the correspondences in causal structure between the story and the problem, particularly with respect to the target solution. Note that the labeled arcs are equivalent to higher-order predicates that embed numbered propositions as arguments.

The representation in Fig. 1 can be used to highlight a variety of properties of analogy, as well as some of the issues we have glossed over in constructing this representation. Fundamentally, an analogy consists of a

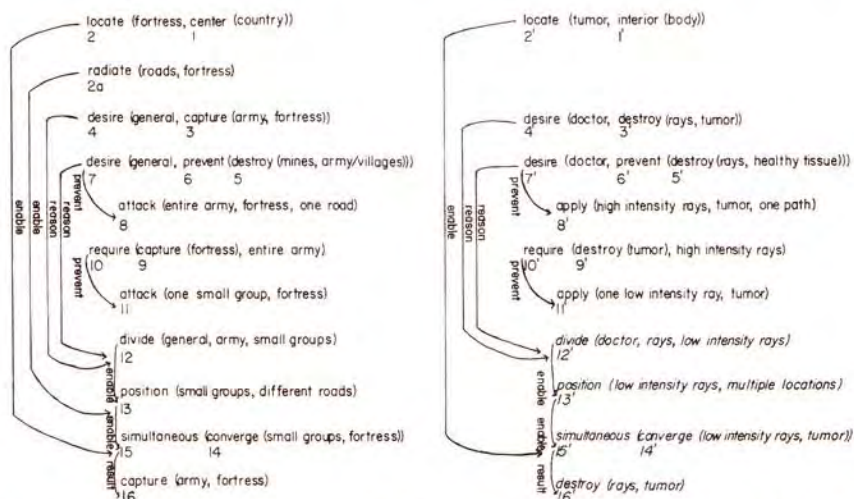


FIG. 1. Analogical correspondences between the Attack-Dispersion story and the radiation problem.

mapping between two sets of propositions. Propositions are matched on the basis of similarity between the corresponding relations. Note that similarity need not imply identity. There need only be a consistent transformation that maps one set of relations onto the other. Boden (1977) gives the example of the Black Mass, which is based on systematic semantic reversals of the Catholic ritual. In the case of our story and problem, we wrote the summaries to maximize the similarity of the relations. Yet dividing an army (proposition 12), for example, is clearly not quite the same as dividing high-intensity rays (proposition 12'). To generate this aspect of the parallel solution the person would need to take account of the relevant differences between an army and rays, perhaps explicitly introducing multiple machines that each emit low-intensity rays. In order to map proposition 1 onto 1' a person must have semantic knowledge of the relation between the meanings of "center" and "interior." Incidentally, note that we assume proposition 1' is included in the macrostructure for the problem as an inference based on knowledge of where the stomach is located. In the case of propositions 16 and 16', we assume the person will use semantic knowledge to transform the relation of "capturing" into the relation of "destroying," operating on the common semantic core (i.e., "overcoming") that links the two relations.

As Fig. 1 indicates, there is clearly a high degree of correspondence between the propositions of the story and of the problem. At the same time, the systems are not perfectly isomorphic. It is probably the case that analogies used to guide problem solving are generally incomplete in some respects (Gentner, Note 1). For example, proposition 2a in the Attack-Dispersion story, which states that many roads radiate outward from the fortress, has no parallel in the statement of the radiation problem. Note that in the story this proposition serves as an important enabling condition for the solution. The absence of any explicit mention in the radiation problem of multiple paths to the target object would presumably hinder generation of the dispersion solution.

For the above example it is plausible to argue that people must infer the fact that there are multiple potential "routes" to the tumor in the course of generating the dispersion solution, even though no such inference is represented in Fig. 1. But in addition, Fig. 1 reveals at least one clear disanalogy between the story and the problem. In a complete analogy, there is a consistent mapping between pairs of arguments. That is, wherever argument A occurs in one relational system, argument A' occurs in the other. For example, in Fig. 1 the role the fortress plays in the story consistently maps onto the role the tumor plays in the problem. Note that the role of the army usually corresponds to that of the rays. However, this is not the case in propositions 5 and 5'. In the Attack-Dispersion story, sending the entire army down one road will result in destruction of the

army (as well as neighboring villages) by *mines*; whereas in the radiation problem applying high-intensity rays to the tumor will result in destruction of the *healthy tissue* by the *rays*. In other words, the army and the rays do not fill corresponding semantic roles in propositions 5 and 5'; rather, the army is the *object* of the process of destruction in 5, while the rays are the *instrument* of the destruction in 5'.

This example illustrates that degree of analogy in part depends on the level of abstraction at which the analogy is defined. In macrostructures slightly more abstract than those depicted in Fig. 1, the fact that an attack by the entire army is impossible would map onto the fact that direct application of high-intensity rays is impossible. At this level the roles of the army and of the rays correspond appropriately. However, the story and the problem are disanalogous at the more specific level depicted in Fig. 1 (i.e., the level of the *reasons* why the two respective courses of action are blocked). This observation suggests that for use in solving a problem the optimal level of abstraction for representing an analogy may be that which maximizes the degree of correspondence between the two relational systems. In many cases a very detailed representation will include disanalogous relations, while a very abstract representation will omit information about important correspondences.

The Process of Analogical Problem Solving

So far we have been discussing how analogical relations may be represented; we must now consider how this information might be used to generate a solution to a problem. For our story analogy and target problem the solution process appears to require three major steps.

(1) A representation of the story analogy and of the target problem (i.e., its initial state and goal state) must be constructed, as described above.

(2) The representation of the story must be mapped onto that of the problem. If the story and the problem are drawn from remote domains, as in our example, the correspondences between arguments will not be immediately obvious. We would therefore expect the mapping process to be initiated by detection of similar relations in the two systems. For example, the person might notice that propositions 2 and 2' both involve location. Accordingly, a mapping between the two propositions will be established. This will automatically establish a mapping between the corresponding arguments (i.e., the fortress and the tumor, the center of the country and the interior of the body). Once a few such correspondences have been detected, the mapping process may proceed in a more "top-down" manner, guided by expectations that previously mapped arguments will continue to play parallel roles. For example, having mapped propositions 2 and 2', the person might assume that 8 maps onto 8' because the role of the fortress should correspond to that of the tumor.

(3) Finally, the person must use the mapping to generate the parallel solution to the target problem. This can be done by constructing a set of solution propositions for the target problem that correspond to the solution propositions of the story. For example, consider how proposition 12' might be generated on the basis of proposition 12. The mapping process will have identified the general with the doctor and the army with the rays. Accordingly, "doctor" and "rays" will be used to fill the argument slots corresponding to "general" and "army." In addition, the relation between the general and the army in 12 will be used to construct a parallel relation between the doctor and the rays in 12'. Thus the idea of the general dividing the army into several small groups will be transformed into the idea of the doctor dividing the rays into a number of low-intensity rays, initiating the dispersion solution to the radiation problem.

ISSUES AND EXPERIMENTS

A number of important questions arise within the framework of the process model we have outlined. A major issue, which we touched upon earlier, concerns the level of macrostructure at which the mapping process takes place. At one extreme the solution process might amount to abstracting a solution principle from the story and then applying it to the target problem. At the other extreme subjects might map the correspondences between the story and the problem at the most detailed possible level. It is possible, of course, that the mapping process may actually proceed partially in parallel on several different levels.

Even at a single level of macrostructure, there may be strategic variations in the degree of mapping that takes place during the solution process. For example, subjects need not derive the entire set of correspondences outlined in Fig. 1 in order to generate the dispersion solution. One possibility, which we will term the "solution-focusing" strategy, is that subjects attempting to apply the story analogy will immediately identify the solution propositions of the story. By doing the minimal amount of mapping required to match the arguments in these propositions with arguments in the radiation problem, the parallel solution could be generated. Subjects using the solution-focusing strategy might thus solve the target problem without entirely grasping the correspondences between the problem statements in the story and in the radiation problem.

Given the lack of empirical research on analogical problem solving, even more basic issues arise. We have sketched a model of how in principle a problem might be solved on the basis of an analogy. However, we do not know whether subjects could actually execute this kind of process for our story analogies and target problem. There seem to be at least three distinct ways in which subjects who have a relevant analogy available might nonetheless fail to derive the parallel solution to a target problem.

The first and most basic is that subjects might be unable to successfully apply the story analogy even if they tried. Second, even if a story analogy is potentially useful, subjects might be unable to locate it in memory, especially if it had been encoded in the context of irrelevant distractor stories. Third, subjects might be able to retrieve a potentially useful analogy and yet fail to spontaneously notice its relevance to the target problem.

The experiments reported below were designed to explore these and related issues. In Experiments I–III subjects were presented with story analogies and given a hint to use them to solve the radiation problem. In addition to investigating whether subjects can in fact use analogies to generate problem solutions, Experiments I and II were intended to provide some information about the processes involved in analogical problem solving. In Experiment I the oral protocols of subjects solving the target problem were analyzed, and in Experiment II the degree of analogical correspondence between the story and the target problem was varied. In Experiment III subjects were asked to first solve the problem presented in the story themselves (rather than having its solution presented to them), and then to attempt to use their solutions as aids in solving the target problem. Experiments IV and V investigated the ability of subjects to solve the target problem on the basis of story analogies stored in memory. In addition, the latter two experiments varied whether or not subjects were provided with a hint to use a story analogy to help solve the target problem. These experiments thus examined the propensity of subjects to spontaneously notice and apply potential analogies between remote problem domains.

EXPERIMENT I

Experiment I was designed to demonstrate that subjects can use an analogy from a remote domain as a hint for solving a problem. Subjects first read a story analogy, and then attempted to propose as many solutions as possible to the radiation problem. By varying the nature of the solution suggested by the story, we hoped to influence the likelihood that subjects would generate specific solutions to the target problem. Subjects' "thinking aloud" protocols were tape recorded and later analyzed as a source of evidence regarding the process of analogical problem solving.

Subjects in three experimental conditions read one of three stories about a military problem and its solution (see Appendix I). Table 2 informally illustrates the correspondences among the three stories and the radiation problem. The statement of the radiation problem (see Introduction) was worded so as to minimize obvious lexical or syntactic correspondences with the story analogies. The Attack-Dispersion, Open Supply Route, and Tunnel stories all have identical first paragraphs describing

TABLE 2
Schematic Outline of Duncker's Radiation Problem Showing
Correspondences with Analogous Stories

Problem Statement	Radiation problem	Story analogies—Experiment I
Problem setting	Doctor has rays. Patient has tumor. Tumor in stomach, surrounded by healthy tissue.	General has army. Country has dictator. Dictator in fortress in center of country, surrounded by villages. Roads radiate from fortress like spokes on a wheel.
Desired goal	Destroy tumor with rays.	Capture fortress with army.
Problem constraints	High-intensity rays destroy tumor but also destroy healthy tissue. Low-intensity rays destroy neither tumor nor healthy tissue. Impossible to operate.	Entire army can capture fortress, but large group detonates mines on roads, destroying army and villages. Small group of men can pass safely over roads but can not capture fortress.
Solutions		
Type I	Reduce intensity of rays on way to tumor.	Reduce size of group traveling to fortress on one road.
Dispersion (Attack-Dispersion story)	(1) Many low intensity rays (2) From different directions (3) Simultaneously	(1) Many small groups of men (2) From different directions (3) Simultaneously
Type II	Avoid contact between rays and healthy tissue.	Avoid contact between army and mines.
(1) Open passage (Open Supply Route story)	Send high-intensity rays through an open route, (e.g., esophagus).	General discovers road that is not mined, and sends entire army down this road.
(2) Operation (Tunnel story)	Make an incision in stomach wall, removing healthy tissue from path of rays, and apply high intensity rays to tumor. ^a	Dig tunnel under mines, and send entire army through.
Resulting goal state	Radiation of high-intensity reaches tumor. Tumor destroyed. Healthy tissue intact.	Entire army reaches fortress. Fortress captured. Army and villages preserved.

^aIncision violates constraint.

the problem setting, desired goal, and the constraints on a solution. These aspects of the stories are analogous to the radiation problem, as discussed earlier (see Fig. 1).

However, the stories differ in their second paragraphs, which state the

general's solution to his problem. In the Attack-Dispersion story (described in the Introduction) the general divides his army into small groups and sends them simultaneously down different roads to the fortress. The analogous solution to the radiation problem is the "dispersion" solution: have multiple low-intensity rays converge at the tumor. This is a very effective solution,² but one which subjects seldom generate spontaneously. Duncker (1945) reported that only 2 of 42 subjects arrived at this dispersion solution, and both were prompted by the experimenter. A basic difficulty that appears to block generation of this solution is that people do not spontaneously think of rays as having the property of "divisibility." In fact, Duncker found that the frequency of the dispersion solution increased when the term "particles" was substituted for "rays" (presumably because particles are more obviously divisible).

In the Open Supply Route story the general discovers an unblocked road leading to the fortress, and sends the entire army down this open road. An analogous radiation solution is to direct high-intensity rays down the esophagus (or some other open passage, such as the intestines) to the stomach. This solution was generated relatively frequently by the subjects tested by Duncker (29% gave the open passage solution as opposed to only 5% who gave the dispersion solution). In the Tunnel story the general digs an underground tunnel and sends his army through it to the fortress. Analogous radiation solutions might be to operate to expose the tumor to the rays, or to insert a tube through the stomach wall and send rays through it to the tumor. Many of Duncker's subjects (40%) spontaneously suggested such solutions. However, such procedures to create an open route to the tumor involve operating, and hence conflict with one of the constraints imposed on the radiation problem (that it is impossible to operate). The Tunnel story is therefore a kind of "false analogy" to the radiation problem. That is, although the problem statements are analogous, the solution suggested by the story is inappropriate. If the analogy is nevertheless applied, subjects given the Tunnel story might be especially likely to momentarily disregard the problem constraints and propose an operation solution to the radiation problem.

Although the above analysis of the analogous relationships between various solutions to the military problem and to the radiation problem was

² This solution is functionally very similar to the standard medical procedure for radiation therapy, which is to rotate the radiation source around the patient (or vice versa) in such a way that the tumor is always the focal point of the radiation. The malignancy thus receives a cumulative dose of radiation while other areas receive a lesser amount. One difference is that our dispersion solution involves simultaneous application of the rays, whereas the medical procedure takes advantage of the fact that the effects of radiation summate over time. Even knowledge of the medical procedure would therefore be unlikely to lead to the exact solution corresponding to the Attack-Dispersion story.

initially based on the experimenters' intuitions, we will see below that subjects' ratings essentially confirm the validity of this analysis.

The primary prediction in Experiment I was that each story analogy would tend to increase the frequency of the analogous solution to the radiation problem, relative to the solution frequencies obtained for control subjects given no prior story. However, there are additional ways in which the story analogies might influence the solutions given to the target problem. First, note that the problem statements for all three stories contain all the enabling conditions (see Fig. 1) for generating the dispersion solution (e.g., the central location of the fortress, the roads radiating outward, the fact that small groups can travel on the roads). Accordingly, subjects might spontaneously think of the dispersion solution to the general's problem, and then use it to generate the parallel solution to the radiation problem. If so, subjects given the Open Supply Route and Tunnel stories might also produce the dispersion solution more often than would control subjects.

It is also possible that giving subjects a story analogy may actually hinder the generation of nonanalogous solutions. That is, attempting to generate a parallel solution to the target problem may create a kind of "set" effect, so that other possible solutions (e.g., immunizing the healthy tissue to protect it from the rays) will not be discovered. If such a set effect is obtained, control subjects should produce more total solutions than experimental subjects, and in addition there should be qualitative differences between the solutions produced by control subjects versus subjects given story analogies.

Method

Subjects were divided into four conditions, each receiving either the Attack-Dispersion story, the Open Supply Route story, the Tunnel story, or no story (the control group), prior to solving the radiation problem. The control condition used no story at all, rather than an irrelevant story, because it seemed possible that an irrelevant story would actually interfere with the solution process. (A control condition using an irrelevant story was included in Experiment V below.) Subjects were tested individually by a trained experimenter and sessions were tape recorded. All subjects first solved Duncker's (1945) "candle problem" to familiarize them with the process of thinking aloud while problem solving. Subjects were told that the problems required some creativity for their solutions, and that they should not feel inhibited about making any suggestions that came to mind. They were also encouraged to give an ongoing account of what they were thinking about. Subjects were asked to begin by reading the problem out loud in order to get them used to speaking in front of the experimenter.

Following the practice problem subjects in the experimental conditions were told that they would receive two further problems, the first of which would also have a solution given for it. They were told to read the first "story problem" aloud and then to orally summarize the gist or point of the story. This part of the procedure was omitted for subjects in the control condition. All subjects then read the radiation problem and began to solve it. They were reminded to talk out loud, and encouraged to interrupt their reading of the problem at any

time if a solution occurred to them. Experimental subjects were told to try to use the first story problem as a hint in solving the second (radiation) problem. However, they were also told that it was not necessary to use the prior story in order to solve the problem. Subjects were allowed to reread the story analogy at any time.

As subjects worked on the radiation problem the experimenter was prepared to intervene with an explicit hierarchy of prompts. If subjects were not explicit about the nature of a proposed solution, the experimenter asked them to clarify it, sometimes by drawing a diagram. Subjects in the experimental conditions who at first failed to generate the analogous solution were eventually prompted to reread the instructions. If they still did not produce the analogous solution, they were then reminded to use the prior story as a hint.

At the end of the session, subjects were asked to rate each of their proposed solutions on two 7-point scales, as to how "creative" and how "practical" the solutions are. A rating of 1 indicated maximum creativity or practicality.

Forty undergraduates enrolled in introductory psychology at the University of Michigan served as subjects as part of a course requirement. Ten subjects were assigned to each of the four conditions.

Results and Discussion

Frequencies of analogous solutions. Subjects' protocols for the radiation problem were transcribed and scored for the presence of various types of proposed solutions, by two independent scorers. For this purpose any suggestion, even if it was eventually rejected by the subject, was counted as a proposed solution. The results of major interest concern the three types of proposals that are analogous to the solutions embodied in the story analogies—the dispersion solution, the open passage solution, and operation solutions. Table 3 presents the percentage of subjects in each condition who produced these various types of proposed solutions. The frequency of each solution was highest for subjects who received the relevant story analogy, i.e., the dispersion solution was most frequent for the Attack-Dispersion condition, the open passage solution was most frequent for the Open Supply Route condition, and operation solutions were most frequent for the Tunnel condition.

These differences in solution frequencies were most dramatic for the dispersion solution. All 10 subjects who were given the Attack-Dispersion story produced this solution, whereas not a single control subject did so. For this solution the frequency differences among the four conditions were highly significant, $G^2(3) = 33.9$, $p < .001$, as was a comparison between the Attack-Dispersion condition versus all others, $G^2(1) = 30.9$, $p < .001$.³

The frequencies of the open passage solution and of operation solutions were also influenced by the story analogies. Seventy percent of subjects in the Open Supply Route condition produced the open passage solution, as opposed to 20% of subjects in all other conditions, $G^2(1) = 8.21$, $p <$

³ All contingency table analyses reported in the present paper use the G^2 statistic (maximum likelihood χ^2) (Bishop, Fienberg, & Holland, 1975).

TABLE 3
 Percentage of Subjects in Each Condition of Experiment I Who
 Proposed Various Solutions to the Radiation Problem

Condition	Proposed solution		
	Dispersion	Open Passage	Operation ^a
Attack-Dispersion story	100	10	30
Open Supply Route story	10	70 ^b	50
Tunnel story	20	30	80
Control	0	20	50

^a This solution type includes proposals to operate to clear a path for the rays or to insert a tube through the stomach wall and send rays through to the tumor.

^b This value includes one subject who proposed the abstract idea of finding an open passage through which the rays could be directed at the tumor, but who failed to suggest the esophagus or any other concrete possibility as a route.

.01. Eighty percent of subjects in the Tunnel conditions produced operation solutions, as opposed to 43% of subjects in all other conditions, $G^2(1) = 4.29, p < .05$.

Ratings of stories as solution prompts. As we noted earlier, we initially used our own intuitions about the semantic parallels between the military and the medical solutions to predict the radiation solutions that would be triggered by the various story analogies. In order to assess whether subjects shared our intuitions we administered a rating task to 51 undergraduates, none of whom previously knew of any solutions to the radiation problem. For each of the three stories used in Experiment I, 17 subjects first read the story, then the radiation problem, and finally the six possible solution proposals listed in Table 4. Subjects rated each of the proposals (which were listed in a random order) on a 9-point scale as to how likely they thought it was that the story they had just read would make them think of the proposed solution. A rating of 9 indicated maximum likelihood that the story would suggest the solution. Subjects were instructed not to consider the practicality of the various solutions in making their judgments.

The mean ratings are presented in Table 4. Note that proposal 1 is the dispersion solution, 2 is the open passage solution, and 3 and 4 are operation solutions. Proposals 5 and 6 are two other fairly common suggestions that we expected would be relatively unrelated to all of the three stories. (Proposal 5 is a type of "accumulation" solution, which will be discussed in connection with Experiment III). The proposal ratings differed greatly depending on which story subjects had read, $F(10,240) = 8.84, p < .001$. For each story analogy, Newman-Keuls tests were performed on the differences among the ratings for the six proposals. In the case of the

TABLE 4
Mean Ratings of Likelihood that Stories Would Help to
Think of Various Radiation Solutions^a

Story	Proposed solutions					
	1	2	3	4	5	6
Attack-Dispersion	8.41	2.53	3.71	2.94	4.88	3.47
Open Supply Route	3.59	6.65	5.82	4.58	2.94	2.29
Tunnel	5.12	4.47	6.47	5.35	2.71	1.82

^a A rating of 9 indicated maximum likelihood that the story would suggest the solution.

Attack-Dispersion story, subjects gave higher ratings to the dispersion solution than to any other ($p < .01$). For the Open Supply Route story, the open passage solution received the highest rating, although it did not differ significantly from the ratings given the two operation solutions. However, these three solutions were all rated significantly higher than any of the others ($p < .05$). For the Tunnel story, the "insert a tube" solution (proposal 3) received the highest rating, followed by the other operation solution (proposal 4), the dispersion solution, and the open passage solution. These four proposals did not differ significantly from each other. However, the two operation solutions received significantly higher ratings than proposals 5 and 6 ($p < .05$).

The fact that subjects gave the highest rating of all to the Attack-Dispersion story as a prompt for the dispersion solution is consistent with the fact that this story appeared to be the most effective analogy in Experiment I. The overlap in the ratings for the solutions analogous to the Open Supply Route and Tunnel stories (proposals 2, 3, and 4) also reflects trends in the observed solution frequencies (see Table 3). Among the three story conditions, the Open Supply Route condition produced the second highest frequency of operation solutions, while the Tunnel condition produced the second highest frequency of open passage solutions. Notice that both types of solutions involve avoiding contact between the rays and the healthy tissue, by directing the rays through an unobstructed

route. The basic difference between the two solution types is simply that the open passage solution makes use of a preexisting route (the esophagus), while the operation solutions require that the route be constructed (by some type of operation).

At first glance the relatively high rating given to the Tunnel story as a prompt for the dispersion solution is more puzzling, since digging a tunnel seems very disanalogous to dispersion of rays. In fact, however, as Table 3 indicates, two subjects in the Tunnel condition actually did produce the relatively rare dispersion solution. Furthermore, as we will see when we discuss the protocols in more detail below, both of these subjects also spontaneously suggested that the general might have sent his army down multiple roads. Recall that the identical first paragraphs of all three stories contain the enabling conditions for the dispersion solution. However, in the Open Supply Route story the given solution involves use of only one road, which may create a set effect that blocks consideration of how multiple roads might be used. In contrast, the solution given in the Tunnel story avoids use of the roads altogether. Since some kind of attack by road is nonetheless an obvious possibility, the stated enabling conditions may lead subjects who read the Tunnel story to think of the alternative solution of using multiple roads, which in turn could prompt the dispersion solution to the radiation problem. While some aspects of the above analysis are certainly speculative, it seems clear that subjects in the rating task were highly sensitive to factors that actually influenced the effectiveness of the story analogies in Experiment I.

Frequencies of other solutions. The possibility of a set effect, mentioned above, raises the general issue of whether story analogies actually block generation of qualitatively different solution proposals. To investigate this question an analysis of variance was performed on the total number of proposed solutions, other than the primary analogous solution, that were given by subjects in the various conditions. That is, dispersion solutions were excluded for the Attack-Dispersion condition, open passage solutions were excluded for the Open Supply Route condition, operation solutions were excluded for the Tunnel condition, while none of the above solutions were excluded for the control condition. In addition, a small number of proposals (a mean of 0.4 per subject) that did not involve use of the rays at all (e.g., Laetrile treatment) were excluded for all conditions.

The average number of nonanalogous solutions produced was 1.10 for the Attack-Dispersion condition, 1.60 for the Open Supply Route condition, 2.70 for the Tunnel condition, and 2.00 for the control condition, $F(3,36) = 4.17$, $p < .025$. Newman-Keuls tests revealed that only the difference between the Tunnel and Attack-Dispersion conditions was significant ($p < .01$). As we noted earlier, subjects in the Tunnel condition

tended to produce a relatively high number of open passage and dispersion solutions. The rating results (Table 4) also indicated that the Tunnel story is a moderately effective prompt for several different solutions. The trends toward fewer alternative proposals in the Attack-Dispersion and Open Supply Route conditions, compared to the control condition, at least suggest the possibility of some kind of set effect.

In order to obtain further evidence regarding a possible set effect, the frequencies of specific radiation solutions, other than those analogous to the various stories, were tabulated for each condition. The solutions examined were proposals to treat the healthy tissue directly, rather than altering the route that the rays take. Specifically, these solutions suggested decreasing the sensitivity of the healthy tissue to the rays (e.g., by a chemical injection, or building up a tolerance to the rays), or covering the healthy tissue with a barrier to protect it from the rays (e.g., by inserting a lead shield to protect the healthy tissue). Such solutions were produced by 30% of the subjects in the control condition, 10% of the subjects in the Tunnel condition, and none of the subjects in the Attack-Dispersion and Open Supply conditions. While the numbers involved were too small to be statistically reliable, these results suggest that an analogy may tend to block generation of alternative types of solutions.

Practicality and creativity ratings. The practicality and creativity ratings that subjects gave for their own solutions were examined. Within each story condition, the ratings given to the analogous solution were compared to the means of the ratings given to other solutions by the same subjects. In the Attack-Dispersion condition seven subjects produced and rated the dispersion solution and at least one other proposal. The dispersion solution was rated as much more practical than other solutions (1.43 versus 4.64), $F(1,6) = 27.2$, $p < .01$. However, the creativity ratings did not differ significantly between the dispersion solution and other proposals (3.43 versus 3.64), $F < 1$. It seems likely that subjects did not perceive their dispersion solutions as especially creative because they were aware of using the prior story to generate an analogous solution.

Parallel analyses for the Open Supply Route and Tunnel conditions revealed no significant differences. Collapsing over subjects in all conditions, the dispersion solution tended to be rated as most practical (a mean of 2.0), followed by the open passage (3.9) and operation solutions (5.4).

We also asked an independent group of 20 undergraduates (none of whom previously knew of the radiation problem) to choose which solution was more practical: the dispersion solution ("apply low-intensity rays from several different directions so they simultaneously converge at the tumor"), or the open passage solution ("send high-intensity rays down the esophagus so they strike the tumor"). Fifteen of the twenty subjects selected the dispersion solution as more practical ($p < .05$ by a sign test).

When asked to justify their decision, 11 of these 15 mentioned the problem of preventing the rays from destroying the tissue lining the esophagus. It may be that some subjects in Experiment I implicitly generated the open passage solution (or an operation solution) on the basis of the relevant story analogy, but then failed to mention it (despite the instruction to mention any possibility) because they recognized its inadequacy. We will consider other sources of difficulty in using the Open Supply Route and Tunnel stories below, when subjects' protocols are discussed in more detail.

Problem-solving protocols. The results discussed so far demonstrate that story analogies can play a major role in directing the problem-solving process. However, they reveal little about the process by which subjects arrive at an analogous solution. We therefore supplemented the quantitative analysis of solution types, reported above, with a more qualitative analysis of subjects' problem-solving protocols. Several aspects of the protocols were examined. Occasions when the experimenter prompted the subjects to use the story were noted, as were correspondences between the story and the target problem that subjects mentioned in the course of generating solutions. This analysis was, of course, constrained by the overall quality and quantity of the protocols. For example, some subjects insisted that talking aloud hindered their thinking, and consequently did not say very much. Rather than presenting an exhaustive analysis of all the protocols, we will therefore concentrate on particularly suggestive excerpts. While this type of protocol analysis has obvious limitations, it may at least provide some hints about the process of analogical problem solving, and in fact served in part to motivate subsequent experiments.

A major issue, raised earlier, concerns the degree of mapping subjects perform in the process of generating an analogous solution. Do subjects make use of detailed correspondences between the story and the target problem, or do they focus directly on the solution embedded in the story and attempt to apply it to the target problem? Of the 10 subjects in the Attack-Dispersion condition, 7 produced the dispersion solution without any prompt, and 3 produced it after being prompted to refer back to the story. In some respects the protocols for prompted subjects are potentially more informative, since what such subjects say is more likely to reflect an ongoing solution process, rather than the result of a process already completed. The protocols of 2 of the 3 prompted subjects suggested use of a solution-focusing strategy.

Table 5 presents an excerpt from the protocol of one of these subjects, S15. After the prompt to use the story, this subject clearly focuses on the solution of dividing up the army into groups and immediately generates the parallel solution to the radiation problem. There is no apparent map-

TABLE 5
Portion of Protocol for S15 (Attack-Dispersion Condition)

Subject reads radiation problem.

S: Alright I, what I most, what I'd probably do is send in the ray at sufficiently high intensity and then taking the risk that the tissues, the healthy tissues that would be destroyed, could be repaired later on. Trying to relate this to the other problem, I could say that you could give multiple treatments of low-intensity ray. But from this problem it seems that they won't have effect on the tumor so . . . so I don't think that would work.

Later . . .

E: Okay. And as a last question can you give me a, tell me ways in which your solution would satisfy the constraints of the experiment?

S: What are the constraints of the experiment?

E: Okay, i.e., that the healthy tissue will not be destroyed, and the tumor will be?

S: Alright, in that way my first suggestion would probably not be the way to go at it. Because that way you're getting low intensity so it won't destroy the tissue and hopefully over a period of time the additive effect of low-intensity rays would kill the tumor. But from reading the article, I don't know if that would work or not, because it says that a low-intensity ray doesn't have any effect on the tumor at all. So I don't know. I don't know any other possible ways of doing it.

E: Would it help to possibly go back to the story and see whether you can apply that?

S: Well, that's what I was trying to do here. It says here he divides his army into different small groups. Okay, may . . . possibly. What they could do, but this is a whole new solution now, possibly what they could do is attack the tumor from a multiple of directions with lower intensity rays and then, since you're coming in from all different directions, the healthy, with small-intensity rays you're not going to be destroying the healthy tissue but you're, and they'll all converge at the point of the tumor which will hopefully destroy the tumor.

ping between the initial problem stated in the story and the target problem.

Notice also that the solution S15 proposes prior to the prompt involves the idea of applying many low-intensity rays. After the prompt, the subject produces the dispersion solution by augmenting this aspect of the earlier solution with the idea of sending rays from many angles. This pattern of gradual solution development was also evident in the protocol of another prompted subject in the Attack-Dispersion condition. In such cases it appears that the subjects were working in the appropriate direction (in the sense of Maier, 1930) prior to producing the complete dispersion solution. That is, at first they seemed to have the abstract idea that the solution should involve reducing the intensity of the rays on the way

to the tumor, but only later were able to develop a concrete solution satisfying the problem constraints. S15 appeared to use the story analogy to develop the early partial solution; however, other subjects apparently generated an abstract partial solution independently of the analogy.

The details of the problem-solving process are less evident in the protocols of the seven unprompted subjects, since they expressed the solution all at once. Three of these subjects simply stated the solution and alluded to the usefulness of the prior story (saying, for example, "considering the problem before"). These subjects did not mention any specific correspondences between the story and the target problem, and hence their protocols were quite unrevealing with respect to the solution process.

However, two other unprompted subjects did spontaneously mention correspondences between the problems. Immediately after reading the radiation problem, S23 stated:

Like in the first problem, the impenetrable fortress, the guy had put bombs all around, and the bombs could be compared to the destruction of healthy tissue. And so they had to, they couldn't go in in mass through one road, they had to split up so as not to destroy the healthy tissue. Because if there's only a little bit of ray it doesn't damage the tissue, but it's all focused on the same spot.

Table 6 presents a portion of the protocol for S28, another unprompted subject. This subject's protocol is particularly interesting because he claimed that he generated the dispersion solution on the basis of an analogy between the radiation problem and an actual problem he had read about the previous night (using lasers to fuse the filament in a lightbulb without breaking the glass). This may be an unanticipated instance of the use of a "real world" analogy to solve a problem. However, when later questioned by the experimenter, S28 clearly was also aware of correspondences between the radiation problem and the prior story. It is therefore uncertain which analogy initially triggered generation of the dispersion solution. Also note that at the beginning of this excerpt S28 remarks that the dispersion solution made it hard to think of alternative solutions, suggesting that he was aware of a set effect. No other subject explicitly mentioned such an inhibitory effect of the prior story.

It is clear in the above two cases that the subjects noticed some correspondences involving the initial conditions and constraints of the story and target problem. However, it is difficult to tell whether these aspects of the mapping process were instrumental in generating the analogous solution, or whether subjects simply mentioned the correspondences to justify the adequacy of the solution, after it had already been generated. In general it was not clear what particular correspondences were central to the solution process. However, several subjects alluded to the importance of the phrase "like spokes on a wheel." Recall that the existence of

TABLE 6

Portion of Protocol for S28 (Attack-Dispersion Condition)

Subject reads radiation problem and states dispersion solution. Experimenter asks for other solutions and subject suggests operating to expose tumor.

S: I like my first solution so much that it's hard to come up with any others.

E: Can you tell me how you arrived at your first solution?

S: To tell you the truth I was thinking about that problem last night.

(Experiment asks for clarification).

S: I remembered reading an ad at one time on one. Some company has really expensive lightbulbs and when the filament breaks inside it's really expensive to replace the lightbulb so what they do is take lasers from all different angles and, like shooting through they don't disturb the glass but when they concentrate they fuse the filament.

E: Oh, I see, very interesting.

S: A . . . other than that I would, I really couldn't, don't think I could come up with other solutions.

E: Okay, can you tell me whether you applied any of the hints from the previous story at all?

S: Yeah, I would say, yeah, the fortress is similar to the tumor and the army is the same as X rays or rays that destroy the tumor, and they cannot all pass through the organism, i.e., countryside, at the same time or they blow up.

multiple routes is a critical enabling condition for the solution embodied in the Attack-Dispersion story, and it has no explicit parallel in the statement of the radiation problem. This aspect of the story analogy may therefore serve to generate the critical insight that it is possible to send rays from multiple directions. One illustrative example is the following excerpt from the protocol of S7 in the Attack-Dispersion condition, which begins immediately after the subject had read the radiation problem:

Well, I already know the answer. I knew it when I read it. I might as well stop and say that. What you do is you have a bunch of rays that are weaker, and you point them so that they concentrate on one point. So you just have many different angles. It could not only be two dimensional, the analogy of the spokes on the fortress. But you could even have it three dimensional, so you could have a whole ball of spokes going in. And you would have a high intensity right at the tumor.

In addition, the protocols of all three subjects in the Open Supply Route and Tunnel conditions who produced the dispersion solution suggested that it was triggered by the idea of multiple converging routes. For example, immediately after S2 in the Open Supply Route condition read the problem, she expressed the idea of using a "circular approach" (which in her earlier story summary she explicitly related to the phrase "spokes on a wheel"). This idea then led to the multidirectional aspect of the dispersion solution to the radiation problem.

Two subjects in the Tunnel condition who produced the dispersion solution did so after first spontaneously remarking that the general might have sent his men down multiple roads. One other subject, in the Open Supply Route condition, also suggested the dispersion solution to the military problem, but failed to apply it to the radiation problem. The use of self-generated solutions to help solve an analogous problem will be investigated more systematically in Experiment III.

Subjects' protocols also provided information about some of the difficulties they encountered in attempting to apply analogies based on the Open Supply Route and Tunnel stories. As we pointed out earlier, the open passage solution is not especially practical and some subjects may have thought of this solution without mentioning it. For example, S32 in the Tunnel condition gave the open passage solution as an afterthought at the very end of the interview, and also outlined the problems with it: that the esophagus is not straight, and that there would be "refraction off the esophageal walls, or absorption of the rays," which would destroy tissue. Three subjects attempted to overcome this difficulty by suggesting that a ray-proof tube through which the rays could be directed should be inserted down the throat.

In addition, the nature of the analogy suggested by the Open Supply Route story is somewhat different from that suggested by the other two stories. The solutions embodied in both of the latter stories suggest procedures that can be used to generate parallel solutions to the radiation problem (dividing the rays in the case of the Attack-Dispersion story, operating in the case of the Tunnel story). In contrast, the Open Supply Route story only suggests that an existing open passage might be used. The subject must then search memory to find a concrete example of such an open passage to the stomach (e.g., the esophagus). Applying the analogy thus involves two steps: mapping the abstract idea of an open passage from the story to the target problem, and then thinking of a concrete example of such a passage. The difficulty of applying the analogy may account for the fact that four of the seven subjects in the Open Supply Route condition who gave the open passage solution had to be prompted to use the story.

Table 7 presents a portion of the protocol for S19 in the Open Supply Route condition. This subject works through a rather detailed mapping of the correspondences between the story and the radiation problem. But while she clearly develops the abstract idea of finding an open passage, she fails in the attempt to think of a concrete example. The partial solution produced by S19 can be contrasted with the complete lack of success apparent in the protocol of S37 in the Open Supply Route condition:

The only thing that is apparent to me is that the general had other information that he knew that one of the thoroughfares would be left open, and so he was able to

TABLE 7
Portion of Protocol for S19 (Open Supply Route Condition)

-
- E: It might help if you reread the instructions here. This part.
(S rereads radiation problem.)
- S: Okay, so what was the first problem? The spokes of the wheel—right?
- E: Right.
- S: So the center fortress deal would be the idea of the tumor. That's . . .
- E: Okay.
- S: And then the spokes that blow up a little would be like the healthy tissue that blows up a little bit. And so with that one the guy had one route that was gonna do it. I guess in this one that's what you have to do is find the one route that would work.
- E: Okay.
- S: And, I think, and not use the other ways.
- E: Okay. What would that be?
- S: That would mean we have to find one approach that was going to get to the tumor without getting the healthy tissue. And I don't see how you could do that. Cause it's not—it doesn't seem like it's the same thing.
- E: What doesn't seem like the same thing?
- S: Well the idea that road, with a road its possible to have one road unguarded but without, in your body there's always going to be, unless the tumor was right on the outside, there would always be some tissue you would have to go through to get to it.
-

use that. But, unless the doctor had some new information or some other treatment, I don't see any other applications from the first problem to the second problem.

Notice that S37 appears to have mapped the story and the target problem at a very abstract level of macrostructure, so that the perceived analogy (the general had new information, so perhaps the doctor might also) was too vague to yield a specific solution proposal for the radiation problem.

In the case of the Tunnel condition four of the eight subjects who generated operation solutions received a prompt to use the story before they did so. Some subjects may have been reluctant to suggest an operation solution because they were aware that it violated a constraint given in the problem statement.⁴ An excerpt from the protocol of S24, presented in Table 8, illustrates the kind of difficulty encountered in this condition. The protocol suggests that the subject was quite carefully mapping components of the story onto components of the radiation problem. However,

⁴ Actually, some subjects pointed out that the problem statement is somewhat vague on this point. The statement that "it is impossible to operate on the patient" might be interpreted as meaning only that it is impossible to operate and remove the tumor, rather than that an operation of any kind is impossible. However, protocols and practicality ratings indicated that most subjects who suggested operating to expose the tumor considered the proposal dubious at best.

TABLE 8
Portion of Protocol for S24 (Tunnel Condition)

-
- E: If you read your instructions, it says that this story might give you some hints . . . What are you thinking?
- S: Well, I remember that the main way they solved this problem was they dug under the fortress and went around it. So, possibly in this situation, you could go around the healthy tissue. I don't know how you'd do that . . . I see an analogy, it's not real clear.
- E: Why isn't it clear?
- S: Because when I picture healthy tissue in my mind, healthy tissue all over and, you know, just like a tumor in between all this healthy tissue. But here, the mines they're on top near the surface of the ground, so they can, you can dig under those and you won't really have any problem. But here no matter where you go, like a circle around the healthy tissue . . . maybe an operation.
- E: Except that one of the constraints of the experiment says that you can't operate.
- S: Okay, that's not possible . . . maybe . . . I was thinking that maybe you could give intervals of high intensity, but I don't know, that still would probably destroy the healthy tissue.
- E: Can you think of anything else? . . . Is this problem, the previous story, is that distracting?
- S: (mumbles) Again, I'm looking for an analogy between the two. And kind of set up the same problem and solve it the same way, but I don't know if I can or not.
- E: So, can you think of any other possibilities?
- S: (long pause) . . . No.
-

the subject was unable to generate a satisfying parallel solution to the target problem.

The overall impression created by the problem-solving protocols is that the generation of analogous solutions involves a conscious process of mapping correspondences between the story and the target problem. The degree of mapping required seems to vary a great deal. Sometimes mapping was done in considerable detail, particularly if the subject was having difficulty producing a parallel solution. In other cases noticing one or two major points of correspondence seemed sufficient to generate the solution. In some instances, particularly for dispersion and open passage solutions, aspects of the solution were clearly generated in sequential steps.

The protocols of control subjects differed in several ways from those of subjects in the story conditions. Control subjects more often were prompted to talk. Sometimes they seemed confused and asked if there really was a solution to the problem. In addition, control subjects received fewer prompts to clarify their solutions. The latter finding raises the question of the extent to which the use of story analogies may depend on verbal or nonverbal feedback from the experimenter. Since a primary concern in Experiment I was to obtain interpretable oral protocols, a

considerable amount of probing by the experimenter was essential. However, it would clearly be desirable to demonstrate that people can use analogies to generate problem solutions without interacting with the experimenter. Experiment II was designed to serve this purpose, and also to provide additional information about the degree of mapping required to produce a solution on the basis of a story analogy.

EXPERIMENT II

In order to avoid any possibility that subjects could be led to a particular solution by the experimenter, Experiment II used a noninteractive procedure in which story analogies and the radiation problem were administered in booklet form. Experiment II was also designed to assess the degree of mapping required to generate a solution on the basis of an analogy. For this purpose different subjects were given one of two matched stories. While both stories embodied the same solution (the dispersion solution), they differed in the degree of correspondence that existed between their problem statements and the radiation problem. If correspondences between the problem statements play an important role in generating a parallel solution to the target problem, lessening the degree of analogy between the story problem and the radiation problem should reduce the probability that subjects will be able to use the story to produce the analogous solution.

Method

Materials. The stories used in Experiment II were the Attack-Dispersion story (version 2) and the Parade-Dispersion story (see Appendix II). Table 9 presents a schematic outline of the Parade-Dispersion story, which can be compared to the outlines of the Attack-Dispersion story and radiation problem (Table 2). The Attack-Dispersion story used in Experiment II is essentially identical to that used in Experiment I. The story was rewritten slightly to match it as closely as possible with the Parade story in length and wording. Note that the setting information is nearly the same in the Attack and Parade stories. In particular, the critical enabling conditions for the dispersion solution (centrally located fortress, multiple roads radiating outward) are present in both stories. In addition, the second paragraphs of the two stories, which describe the general's dispersion solution, are identical (except for the final sentences, which state the goal that has been achieved).

However, the problem statement for the Parade story is substantially different from that for the Attack story, and it is disanalogous to the radiation problem. In the Parade story the general is not trying to attack the dictator in the fortress, but rather to stage a parade that meets the dictator's specifications. The constraint of the mined roads has been removed; and use of the entire army is required not to produce a sufficiently strong assault, but rather to produce a sufficiently impressive display. The problem statements of the Parade story and of the radiation problem are thus disanalogous in several respects. The Parade problem lacks the basic element of "desire to overcome a target object." The basis of the constraint against sending the entire army down one road (the general would lose his rank) does not parallel the basis of the constraint against using high-intensity rays (healthy tissue would be destroyed). Only the setting information and the constraint against sending the full army down one route remain analogous to features of the radiation problem. Accordingly, the mapping process

TABLE 9
Schematic Outline of Parade-Dispersion Story

Problem statement	
Problem setting	General has army. Country has dictator. Dictator in fortress in center of country, surrounded by villages. Roads radiate from fortress like spokes on a wheel.
Desired goal	Produce impressive parade that can be seen and heard throughout entire country.
Problem constraints	Sending entire army down one road fails to produce impressive parade. If parade fails to impress dictator, general will lose his rank.
Solution	
Dispersion	Divide up parade so that each part of country sees part of parade. Use (1) Many small groups of men (2) From different directions (3) Simultaneously
Resulting goal state	Parade seen and heard simultaneously throughout country. General preserves his rank.

subjects can perform in order to use the Parade story to generate the radiation dispersion solution will be limited by the lack of correspondence between the problem statements for the story and the target problem.

As in the Attack story, the general in the Parade story solves his problem by dividing his troops and sending them down multiple roads to the fortress. But in the Parade story the procession of soldiers to the fortress directly constitutes achievement of the goal state, whereas in the Attack story the movement of troops is simply the means by which the final goal (capture of the fortress) is achieved. Furthermore, in the Parade story the fact that the troops converge on the fortress is a more or less incidental aspect of the solution procedure, whereas in the Attack story this aspect of the solution is critical. Thus even though the surface description of the solution is the same in both stories, the solution contexts differ. Such contextual differences may influence the precise interpretation subjects give to the solution.

In order to assess whether the Parade and Attack stories in fact suggest similar solutions to the radiation problem, two independent groups of 17 undergraduates rated the stories as prompts for various radiation solutions. (The Parade story was simply included as an additional condition in the story rating task discussed under Results of Experiment I.) The six solutions that were rated are listed in Table 4. While there was a slight trend for the Attack story to be rated higher than the Parade story as a prompt for the dispersion solution, the two stories did not produce significantly different ratings, either as a main effect, $F < 1$, or as an interaction, $F(5,160) = 1.32, p > .25$. These rating results thus confirm that the two stories, despite the differences in their problem statements, suggest essentially the same solution to the radiation problem. Experiment II was designed to determine whether subjects actually solving the radiation problem would be hindered by a disanalogous problem statement.

Procedure and subjects. Subjects were divided into three conditions, receiving either the Attack story, the Parade story, or no story (control condition) prior to working on

the radiation problem. The test booklet consisted of instructions, story analogy (for experimental conditions), radiation problem, a solution sheet, and a final questionnaire, each on a separate page. Experimental subjects first read the story and then wrote a brief summary of it, referring back to the story if they wished. All subjects then attempted to solve the radiation problem. Subjects in the experimental conditions were told that the first story might give them some hints for solving the test problem, although it was not necessary to use the prior story to solve the problem. They were allowed to refer back to the story at any time. All subjects were instructed to write as many solutions as possible in the order they came to mind. They were asked to write down every idea they considered, even those later rejected. Subjects were told to make their proposals as explicit as possible, but not to be concerned with technical medical considerations. The final questionnaire asked subjects to indicate how helpful the story problem was in solving the radiation problem ("not helpful," "somewhat helpful," or "very helpful"), and in what way it was helpful. Subjects were also asked if they had known the solution to the radiation problem prior to the experiment.

Subjects were 143 undergraduates tested in five introductory psychology classes. Forty-seven subjects served in the Attack-Dispersion condition, 46 in the Parade-Dispersion condition, and 50 in the control condition.

Results and Discussion

The data for one subject in the Parade-Dispersion condition who was familiar with the radiation problem and dispersion solution were discarded. The remaining subjects' solution proposals were analyzed for the presence of various solution types, particularly the dispersion solution. This analysis was done by two independent scorers, each blind to the conditions in which subjects served. Disagreements were resolved by discussion. Unlike subjects in Experiment I, subjects in Experiment II were not prompted to fully explicate their solutions. As a result, a number of subjects produced incomplete versions of the dispersion solution. In order to be scored as a complete dispersion solution, three features had to be present in the proposal: (1) the rays are applied to the tumor from different directions, (2) at low intensity, and (3) simultaneously. A partial solution had to contain at least the first feature, the critical element of dispersion. However, partial solutions might omit features 2 and/or 3. In addition, a partial solution might include reference to some degree of damage to healthy tissue. Presence of the latter feature was taken as an indication that the subject did not entirely understand the implications of the dispersion solution, since the problem statement specified that low-intensity rays are harmless.

The percentage of subjects producing complete or partial dispersion solutions differed substantially across the three conditions, as shown in Table 10. Dispersion solutions were produced by 76% of the subjects in the Attack-Dispersion condition, 49% of the subjects in the Parade-Dispersion condition, and only 8% of the subjects in the control condition, $G^2(2) = 53.1$, $p < .001$. The Attack story produced significantly more dispersion solutions than did the Parade story, $G^2(2) = 7.70$, $p < .01$, while the two story conditions together produced significantly more dis-

TABLE 10
Percentage of Subjects in Each Condition of Experiment II Who Proposed
the Dispersion Solution to the Radiation Problem

Condition	Dispersion solution			N
	Complete	Partial	Total	
Attack-Dispersion story	57	19	76	47
Parade-Dispersion story	31	18	49	45
Control	8	0	8	50

persion solutions than did the control condition, $G^2(1) = 45.5, p < .001$. The Attack story also prompted more complete dispersion solutions than did the Parade story. The four subjects in the control condition who spontaneously generated dispersion solutions gave complete solutions. Collapsing over all conditions, dispersion solutions tended to be produced relatively early in the sequence of proposals given by a subject (i.e., prior to the proposal of median rank, $p < .025$ by a sign test).

Table 11 presents the percentage of subjects in the two story conditions who rated the story as "not helpful," "somewhat helpful," or "very helpful," as a function of whether the subject produced a complete dispersion solution, partial solution, or no dispersion solution. The results obtained with such post-hoc questionnaires must be interpreted with caution, since they may reflect hindsight rather than accurate memories of the solution process. Nevertheless, these rating results are at least a source of converging evidence. Most subjects who produced a dispersion solution rated the story as "very helpful." In contrast, the majority of those subjects in both conditions who failed to produce a dispersion solution rated the story as "not helpful at all." There were no reliable differences between the rating patterns produced by subjects in the Attack versus Parade conditions.

The total number of proposed solutions (other than the dispersion solution) was tabulated for subjects in each of the three conditions. The mean number of proposals was 1.17 for the Attack condition, 1.40 for the Parade condition, and 2.12 for the control condition, $F(2,139) = 12.7, p < .001$. A Newman-Keuls test indicated that only the difference between the control condition and the two story conditions was significant, $p < .01$. This decline in the number of alternative proposals given by subjects prompted to generate the dispersion solution replicated the comparable trend obtained in Experiment I.

As in Experiment I, the frequency of a specific class of alternative solutions (decreasing the sensitivity of the healthy tissue to the rays, or

TABLE 11
 Percentage of Subjects Giving Each Helpfulness Rating as a Function of
 Story Condition and Type of Dispersion Solution Given

Condition	Type of dispersion solution given								
	Complete dispersion solution			Partial dispersion solution			No dispersion solution		
	Not helpful	Somewhat helpful	Very helpful	Not helpful	Somewhat helpful	Very helpful	Not helpful	Somewhat helpful	Very helpful
Attack-Dispersion story	0	22	78	11	22	67	64	27	9
Parade-Dispersion story	0	29	71	0	38	63	57	39	4

somehow protecting the tissue) was tabulated for each condition. Whereas 28% of the subjects in the control condition produced such solutions, only 2% of those in the Attack condition and 16% of those in the Parade condition did so, $G^2(2) = 14.6$, $p < .01$. Subjects in the Parade condition produced more solutions of this type than did subjects in the Attack condition, $G^2(1) = 5.78$, $p < .02$. This result is a significant replication of a trend obtained in Experiment I: the more effective the story analogy is in prompting the analogous solution, the more it inhibits production of alternative, disanalogous proposals.

The basic results of Experiment II are thus extremely clear. First, subjects can readily use story analogies to guide their attempts to solve the radiation problem, even without feedback from the experimenter. Second, the effectiveness of analogies in prompting a specific solution is a matter of degree. In particular, a story with a problem statement analogous to that of the radiation problem (the Attack story) was more likely to trigger the dispersion solution than was a story with a problem statement less related to that of the radiation problem (the Parade story). This was true even though both stories embodied similar setting information and solution statements.

EXPERIMENT III

Experiments I and II demonstrated that subjects can use a story analogy, describing a problem and its solution, to guide generation of an analogous solution to the radiation problem. A natural question is whether subjects could also use their own solutions to the initial story problem to help them solve the target problem. You may recall that Experiment I provided some evidence for this possibility. Three subjects who received stories other than the Attack-Dispersion story spontaneously suggested that the general might have divided his troops and sent them down multiple roads; two of these subjects then went on to produce the dispersion solution to the radiation problem. Accordingly, in Experiment III subjects were first given just the problem statement from the Attack-Dispersion story, and asked to suggest what the general might do to capture the fortress. Subjects were then asked to solve the radiation problem, using their solutions to the initial problem as hints.

Method

The problems were administered in booklet form as in Experiment II. The instructions stated that the subject would have to solve "two verbal problems requiring some creativity for their solutions." The first problem consisted of the first paragraph of the Attack-Dispersion story (version 2, Appendix II), followed by the question "What could the general do in order to capture the fortress?" Subjects were to provide as many possible solutions as they could think of. Following this, subjects proceeded to give solutions to the radiation problem. The instructions stated, "... you may find that the first problem that you solved

gives you some hints for solving the second problem, so you should try and use it if you can. . . . However, it is not necessary to use the first problem in order to solve the second." Subjects were told that they could look back to the first problem and their solutions to it at any time. On the final page of the booklet subjects were asked to indicate to what extent the first story problem helped in solving the ray problem ("not at all," "somewhat," "very much"), and in what way it was helpful. They were also asked to indicate if they had known the solution to the radiation problem prior to the experiment.

The experiment was administered to 46 students in two introductory psychology classes.

Results and Discussion

Data from one subject who indicated prior knowledge of the dispersion solution to the radiation problem were discarded. The frequencies of various solutions to the two problems were tabulated for the remaining 45 subjects by two independent scorers; disagreements were resolved by discussion. As might be expected given the extremely loose constraints imposed by the problem statement, the proposed solutions to the military problem were quite varied. Many of these (e.g., airlifting troops into the fortress by balloon) had no apparent correspondence to any potential solution to the radiation problem. However, 22 of the 45 subjects (49%) produced the gist of the dispersion solution: the general should divide the troops into small groups and send them down different roads.

The major question of interest was whether those subjects who produced the dispersion solution to the military problem would be especially likely to then produce the dispersion solution to the radiation problem. Of the 22 subjects who produced the dispersion solution to the military problem, 9 (41%) subsequently produced either complete or partial dispersion solutions to the radiation problem. Five of these subjects produced a complete solution. In contrast, only 3 of the remaining 23 subjects (13%) produced this solution $G^2(1) = 4.61, p < .05$. Two of these subjects produced a complete solution. The value of 41% is significantly higher than the 8% of subjects who produced the dispersion solution to the radiation problem without any prior story problem (Control condition of Experiment II), $G^2(1) = 10.4, p < .01$.

The above results are correlational in nature, since subjects who arrived at the dispersion solution to the military problem did so of their own accord—no experimental manipulation determined the subjects that would produce the critical solution. It could therefore be argued that subjects who produced dispersion solutions to both problems did so because of some general factor related to problem-solving skills or strategies. However, additional evidence suggests that self-generated solutions to the military problem had a causal influence on generation of the radiation dispersion solution. First, dispersion solutions were produced significantly more frequently in Experiment III (regardless of whether a parallel solution was produced for the military problem) than in the con-

trol condition of Experiment II. Second, the questionnaire results also reflected the influence of the solutions produced for the military problem. Of the nine subjects who generated the dispersion solution to both problems, all but one indicated that the military story was "somewhat" or "very" helpful in solving the radiation problem (one other subject did not respond to this question). In contrast, a majority (58%) of the remaining subjects indicated that the story was not helpful at all. Interestingly, two of the three subjects who produced the radiation dispersion solution without first producing the military dispersion solution indicated that the story was somewhat or very helpful (the third subject failed to answer the question). All three of these subjects had suggested that the general could send small groups in succession to the fortress. This solution may have triggered the idea of dividing forces that is critical to the radiation dispersion solution.

While self-generated solutions to the simpler military problem thus facilitated discovery of the radiation dispersion solution, the degree of transfer was less than perfect. The value of 41% radiation dispersion solutions, based on a self-generated military dispersion solution, is significantly less than the 76% radiation dispersion solutions produced by subjects given a military dispersion solution written by the experimenter (Attack-Dispersion condition of Experiment II), $G^2(1) = 8.25$, $p < .01$. There are a number of possible explanations for this difference. Subjects in Experiment III almost always proposed more than one solution for the military problem (a mean of 3.04). Consequently, their own dispersion solution, if they produced it, was usually embedded in other "distractor" solutions. Subjects who did not systematically consider each of their proposed military solutions with respect to the radiation problem may therefore have sometimes missed the critical analogy. Indeed, some subjects may have simply failed to apply their prior solutions at all when working on the radiation problem.

In addition, it is possible that the self-generated dispersion solutions did not always entirely capture the analogy to the corresponding radiation solution. All of the self-generated dispersion solutions expressed the basic idea of sending small groups down multiple roads. However, in some versions the element of simultaneity of attack was absent or not clearly expressed. In other cases subjects suggested that the small groups would meet and regroup near the fortress prior to initiating an attack. This added feature of delay is disanalogous to the radiation dispersion solution (since low-intensity rays cannot "wait and meet up" near the tumor prior to striking it). Of the 22 self-generated dispersion solutions, 10 either lacked a clear expression of simultaneity or added the element of delay. However, since 4 of these 10 subjects succeeded in generating the radiation dispersion solution, we were unable to find any effect of quality of the

military dispersion solution on subsequent transfer to the radiation problem. Nonetheless, it remains possible that in various ways the subjects' dispersion solutions (which were often written in a cryptic fashion) were incomplete in comparison with our own version of the solution. At any rate, it is clear that further research will be required to assess how problem-solving activity per se affects subsequent transfer of a solution to an analogous problem.

In addition to the dispersion solution, another frequently proposed solution to the military problem was to have the general send successive small groups of men down a single road. This "successive groups" solution appeared to trigger an analogous solution to the radiation problem. We defined proposed solutions to the latter problem that involved spreading out ray applications over time (regardless of the intensity of the rays or whether treatment was intermittent or continuous) as "accumulation" solutions. Of 24 subjects who gave the successive groups solution to the military problem, 10 (42%) suggested an accumulation solution to the radiation problem. In contrast, only 3 of the remaining 21 subjects (14%) suggested an accumulation solution to the radiation problem, $G^2(1) = 4.28$, $p < .05$. Furthermore, 2 of the latter 3 subjects had produced the dispersion solution to the military problem. Their accumulation solutions may therefore have been triggered by a relatively abstract analogy between "dispersion over space" and "dispersion over time." This possibility is supported by the fact that an accumulation solution was rated the second most likely (after the dispersion solution) to be prompted by the Attack-Dispersion story (see Table 4).

This tendency for subjects to generate analogous accumulation solutions is particularly interesting because such proposals are clearly ineffective solutions. Simply distributing ray treatments over time will not overcome the basic problem that the rays will have equal effects on both the tumor and the healthy tissue. Since the instructions to subjects stated that they should write down any proposal they could think of, even if they later rejected it, we do not know whether subjects who gave an accumulation solution actually believed it would work. However, this result at least raises the possibility that a "false analogy" between problems (similar to the Tunnel condition in Experiment I) may foster errors in evaluating a parallel solution to a transfer problem.

EXPERIMENT IV

Our central concern in the experiments reported so far was to determine if people can use an analogy to generate a solution to a target problem, and to investigate how analogical problem solving proceeds. Consequently, we simplified the subjects' task in several important ways. First, subjects were always allowed to reread the story analogy at any

time, so that their performance would not be limited by memory factors. Second, the story was always presented alone, so that subjects would have no problem identifying the relevant analogy. Third, subjects were always explicitly told to try to use the story as an aid in solving the target problem. This hint was quite nonspecific; at no time were subjects told the nature of the analogous relationship between the story and problem. Nevertheless, the hint eliminated the need for subjects to spontaneously notice the possible analogy.

In many cases of everyday problem solving in which an analogy could help, the person would have to spontaneously notice the correspondence between the target problem and some analogous problem, either of which might be stored in memory. The two experiments reported below begin to investigate the effect of such additional processing requirements on analogical problem solving.

Method

In Experiment IV subjects first memorized the Attack-Dispersion story (version 3, Appendix III⁵) in the guise of a story recall experiment, and then went on to work on the radiation problem. In addition, subjects also first memorized two additional "distractor" stories written by the experimenters ("The Wine Merchants" and "The Identical Twins"; see Appendix IV). These additional stories were matched closely in length with the Attack-Dispersion story, and also describe problems and their solutions. However, the two distractor stories were intended to be as disanalogous to the radiation problem as possible.

The experiment was administered in booklet form to small groups of subjects. The initial instructions to all subjects stated that the experiment would have two parts, first a story recall task and then a problem-solving task. For the story recall task, subjects first received one story, and were given 3 min to study it. The stories were then collected, and subjects had up to 15 min to recall the story. They were asked to recall the story in as close to its original form as possible, but to give the gist of it even if they couldn't remember the exact wording. All subjects completed their recall attempts within 15 min. When all subjects in a group were finished, the next story was distributed and the study-recall procedure was repeated. The stories were always administered in the order "The Wine Merchants," the Attack Dispersion story (entitled "The General" for subjects), and "The Identical Twins." The critical story analogy was placed in the middle serial position in order to maximize the difficulty of later using the memorized story to solve the radiation problem.

Following the story recall tasks subjects were given a short (3 to 5 min) break, and then asked to write solutions to the radiation problem. However, subjects were divided into two conditions that received slightly different instructions. For subjects in the "Hint" condition, the instructions on solving the radiation problem included the following sentence: "In solving this problem you may find that one of the stories you read before will give you a hint for a solution of this problem." For subjects in the "No Hint" condition, this sentence was deleted from the instructions. The instructions used in the Hint condition were thus comparable to those used in the earlier experiments, whereas the instructions used in the No Hint condition for the first time did not call attention to the potential relevance of the stories to the target problem.

⁵ The wording of the story was modified slightly in this version to match it with other story analogies we intended to use in a larger experimental design. For various reasons these additional conditions were never actually tested.

After completing their solution attempts subjects were given a final questionnaire. Subjects in the No Hint condition were asked whether it occurred to them "to try and use any of the stories from the first experiment to help solve the ray problem"; and if so, how helpful each of the three stories was ("not at all," "somewhat," "very"), and in what way. Subjects in the Hint condition answered only the latter part of the above question. All subjects were also asked if they had known the solution to the ray problem prior to the experiment.

Twenty-seven undergraduates from the Human Performance Center paid subject pool served as paid subjects. Twelve subjects were tested in the Hint condition and fifteen in the No Hint condition.

Results and Discussion

Two independent scorers were entirely in agreement in identifying dispersion solutions to the radiation problem. For the Hint condition, 11 out of 12 subjects (92%) produced the complete dispersion solution. This percentage is, of course, far higher than would be expected for subjects who did not receive the Attack-Dispersion story (under 10% in both Experiments I and II). In addition, 10 of the 12 subjects indicated that the Attack-Dispersion story was "very helpful," one indicated that it was "somewhat helpful," and one did not answer the question. In contrast, 50% of the subjects indicated that "The Wine Merchants" was "not helpful at all," and 91% (10 of 11 who answered the question) indicated that "The Identical Twins" was "not helpful at all." Subjects clearly had no serious difficulty in identifying the critical story analogy in memory and applying it to generate the dispersion solution to the radiation problem.

However, this picture changes dramatically when the results for the No Hint condition are examined. Whereas 92% of the subjects in the Hint condition produced the dispersion solution, only 20% (3 out of 15) of those in the No Hint condition did so, $G^2(1) = 15.5, p < .001$. Furthermore, 2 of these 3 subjects gave only partial solutions (as defined in Experiment II), and indicated that they did not consider using the stories. It is therefore possible, and in fact rather likely, that only 1 of the 15 subjects spontaneously noticed the critical analogy and successfully applied it to produce the dispersion solution.

In response to the questionnaire, 12 of the 15 No Hint subjects indicated that it had not occurred to them to use the stories. The 3 who said they did use the stories all indicated that the Attack-Dispersion story was "very helpful"; however, only one of these subjects actually produced the dispersion solution. The written comments of the other two subjects who said they used the Attack-Dispersion story suggested they were at most sensitive to some very vague, abstract analogy with the radiation problem. For example, one wrote that the story "showed a unique way to attack a problem using methods that were not very obvious or usual." (Similar comments were given by subjects who rated the Wine Merchants

story as somewhat or very helpful.) It therefore seems that no more than 3 subjects, and perhaps only 1, actually noticed the critical analogy without a hint from the experimenter.

Since subjects were randomly assigned to the two conditions, degree of memory for the critical story should have been equalized across the two conditions. To confirm this, the protocols for the Attack-Dispersion story were scored for gist recall. For this purpose the story was divided into 43 propositions (see Appendix III). This propositional division was made using the procedure outlined by Thorndyke (1977), with the addition that adjectives and prepositional phrases that seemed intuitively important to the story were counted as separate propositions. As anticipated, the mean number of propositions recalled did not differ significantly between the Hint and No Hint groups (32.08 versus 33.53), $t < 1$.

While we did not analyze the recall in detail, a few aspects are worth noting. First, performance appeared quite good in an absolute sense, which is consistent with the fact that subjects were almost invariably successful in using the story to generate a solution to the radiation problem (as long as they were told to try). Second, as many investigators of story recall have reported, some propositions were much more memorable than others. Three propositions were recalled by all 27 subjects. These were proposition 1 ("A small country was ruled by a king") and propositions 5 and 6 ("Many roads radiated outward from the fortress like spokes on a wheel"). As we argued earlier, the latter pair of propositions establish a critical enabling condition for the general's solution to his problem. This high level of recall obtained for setting information causally related to the solution is consistent with the problem-based model of story recall proposed by Black and Bower (in press).

EXPERIMENT V

The results of Experiment IV demonstrated that subjects can identify a relevant story analogy encoded into memory in the context of distractor stories, and can use the analogy to generate a solution to a subsequent target problem. However, when the experimental instructions did not provide a hint that the stories might help to solve the target problem, subjects seldom noticed or used the analogy. This suggests that the process of analogical problem solving is neither automatic nor invariably applied by college students as a conscious strategy. The knowledge acquired in the context of the story recall phase of the experiment seemed to be encapsulated in such a way that its pertinence to the problem-solving task was not recognized.

An important question is whether this type of encapsulation of experience is more or less absolute, or whether there are factors that would make a relevant analogy more likely to be noticed even though it was

initially encoded in a recall context. Experiment V modified the design of Experiment IV in order to examine two such possible factors. First, the total memory load was reduced by eliminating the two distractor stories from the recall phase; and second, in one condition the story analogy was presented *after* subjects had read and begun to work on the radiation problem. The latter condition can be viewed as an experimental analog of a situation in which a person "stumbles upon" relevant information in the course of working on a problem, as is often reported in anecdotes describing the "Eureka" experiences of creative thinkers.

Method

The experiment was administered in booklet form as in Experiment IV. Subjects were assigned to one of three conditions. The initial procedure for the Story First condition was identical to that for the No Hint condition of Experiment IV, except that the recall task involved only the critical Attack-Dispersion story. After working on the radiation problem subjects were asked to indicate whether it had occurred to them to use the story from the first experiment to help solve the problem. If they responded "no," they were then asked to try and use the story to generate additional solutions to the radiation problem. The procedure thus involved three steps: (1) reading and recall of the story; (2) an attempt to solve the radiation problem without a hint to use the story; and (3) a final attempt to solve it with a hint.

The Story Second condition was presented in the guise of an "incubation" experiment. Subjects were told they would be given a problem to work on, then would be interrupted to perform a different task (story recall), and then would again work on the problem. The actual procedure involved four steps: (1) an initial 10-min attempt to solve the radiation problem; (2) reading and recall of the Attack-Dispersion story; (3) a second attempt at the radiation problem, without any hint to use the story; and (4) a final attempt to solve the radiation problem after the hint was given. This procedure thus differed from that for the Story First condition only by the addition of step 1, the initial attempt to solve the problem.

If subjects in the Story Second condition were to produce more dispersion solutions than subjects in the Story First condition immediately after recalling the story, this would suggest that initial exposure to the problem makes it more likely that the person will notice a relevant analogy. However, one might argue that such a result could be interpreted as a real "incubation effect." That is, regardless of the nature of the story presented during the intervening recall task, perhaps simply taking a break from the problem would be sufficient to increase the probability of generating the dispersion solution. To control for this possibility, subjects in the Incubation Control condition received exactly the same procedure as did those in the Story Second condition, except that their recall task used one of the distractor stories from Experiment IV ("The Identical Twins"). Accordingly, any tendency for the Incubation Control condition to produce more dispersion solutions after presentation of the story would presumably be due to a beneficial effect of incubation per se, rather than to use an analogy.

Forty-seven undergraduates from the Human Performance Center subject pool served as paid subjects. Seventeen subjects were assigned to the Story First condition, 20 to the Story Second condition, and 10 to the Incubation Control condition.

Results and Discussion

Table 12 presents the percentage of subjects in each condition who produced the dispersion solution during the various steps of the proce-

TABLE 12
 Percentage of Subjects in Experiment V Who Produced Dispersion
 Solution at Each Step of the Procedure

Condition	Before story	After story (no hint)	After story (with hint)	Never	<i>N</i>
Story First	—	41	35	24	17
Story Second	10	35	30	25	20
Incubation Control	10	0	0	90	10

ture. There was no evidence that the manipulation of presenting the problem prior to the story analogy (Story Second condition) increased the probability that subjects would notice or use the analogy. In the Story First condition, 41% of the subjects gave the dispersion solution on their first attempt following recall of the story; while in the Story Second condition, 35% of the subjects produced this solution immediately after reading the story. One subject in the Story First condition gave a partial solution; all other solutions were complete. For the Story First condition we cannot clearly separate subjects who used the story to produce the solution from those who may have produced it spontaneously (as did those subjects in the Story Second condition who gave the dispersion solution prior to seeing the story). However, of the seven subjects in the Story First group who gave the dispersion solution immediately after story recall, six reported that they used the story to help solve the problem. If we accept these reports at face value, it appears that the percentages of subjects in the two conditions who spontaneously noticed and used the analogy were identical (35% in both conditions). The percentages of all subjects who reported that it occurred to them to use the story were also similar across the two conditions (47% in the Story First condition, 40% in the Story Second condition).

It is clear that in both conditions not all of the subjects who eventually proved able to use the story to generate the dispersion solution did so spontaneously. Collapsing over the Story First and Story Second conditions, 43% of the subjects generated the dispersion solution prior to receiving the hint, while a total of 76% eventually succeeded in producing the critical solution. Note that there was a trend toward a higher percentage of subjects generating the solution without a hint in Experiment V (43%), where no distractor stories were used, than in Experiment IV (20%), where two distractor stories were included in the recall task. However, this trend was not significant, $G^2(1) = 2.64$, $p = .10$. It should be noted that the above comparison confounds number of distractor stories with serial position of the critical story (which was always presented in the middle position in Experiment IV). It is possible that distractor stories

would have a less detrimental effect on the likelihood of subjects noticing the analogy if the critical story were presented last, just prior to the problem-solving task.

As in all previous experiments, the story analogy clearly played a critical role in generating the dispersion solution. Whereas 76% of the subjects in the two experimental conditions eventually produced the dispersion solution, only 10% (one subject) produced it in the Incubation Control condition, $G^2(1) = 15.0, p < .001$. Since the one successful subject in the latter group produced the target solution prior to receiving the story, there was not the slightest suggestion that simply taking a break from the problem was sufficient to stimulate discovery of the dispersion solution.

GENERAL DISCUSSION

The present study provides an experimental demonstration that a solution to a problem can be developed by using an analogous problem from a very different domain. Our results substantiate anecdotal descriptions of the role that analogical thinking may play in creative problem solving, and at the same time provide some information about the mental processes involved in analogical problem solving. The results of Experiments I and II indicated that there is considerable variation in the degree of mapping required to generate an analogous solution. In particular, the intermediate frequency of dispersion solutions produced in Experiment II by the Parade story, which was only partially analogous to the radiation problem, supports two important conclusions about the mapping process involved in analogical problem solving. First, subjects in the Parade condition were much more likely to generate dispersion solutions than were control subjects. Thus it seems that subjects can often generate an analogous solution even though a complete mapping between aspects of the prior story and the target problem is impossible. In such cases it seems that a solution-focusing strategy may be sufficient to produce the parallel solution. Second, the Parade story was not as effective as the more completely analogous Attack story in prompting the dispersion solution. This suggests that subjects can also perform a more detailed mapping between the problem statements of the story and of the target problem, and that these additional correspondences are sometimes critical in determining whether the subject arrives at the analogous solution.

However, the types of correspondences between the two problem statements that are most critical in developing a solution are not entirely clear. Numerous subjects in our experiments commented on the importance of the reference in the story to roads radiating outward "like spokes on a wheel." Intuitively, this phrase seems to elicit a spatial image that represents those essential aspects of the dispersion solution that can be applied to both the military and the medical problems. Even though the

stories and the target problem were always presented verbally in our experiments, the problems essentially describe spatial relationships. Our use of a propositional representation to describe the correspondences between the stories and the radiation problem does not preclude the possibility that some form of analog representation plays an important role in the mapping process. For example, the mapping process may in part depend on interpretive procedures that are applied to a mediating spatial image. Further research is needed to explore the role of spatial representation in analogical problem solving.

It is clear that our understanding of the use of analogies in problem solving remains severely limited in many important respects. We certainly need to be cautious in generalizing on the basis of the present study, which used only one target problem and a very limited set of story analogies. While it seems reasonable to expect that comparable results would be obtained with other ill-defined "insight" problems, for which a solution hinges on a small number of critical inferences, this remains to be demonstrated.

It is still less clear whether analogies can be used in a similar fashion to help solve more "computational" problems, for which the solution consists of a series of discrete steps. Reed et al. (1974) were unable to demonstrate positive transfer between two homomorphic "river crossing" problems, except when the correspondences between the arguments of the two problems were described to subjects. In addition, most subjects in the Reed et al. study reported making little or no use of the first problem when solving the second. It is possible that the mapping process required in such multimove problems places excessive demands on memory capacity. However, various procedural differences make it difficult to directly compare the Reed et al. results to those obtained in the present study. For example, subjects in the Reed et al. study solved two successive problems, while in our experiments the solution to the first problem was described in the context of a story (except in Experiment III, in which the probability of transferring the analogous solution was somewhat reduced when subjects solved the first problem themselves). In addition, it is possible that people are able to use analogies more easily in solving some computational problems than in solving others. For example, Hayes and Simon (1977) have demonstrated positive transfer between isomorphic versions of the Tower of Hanoi puzzle, another computational problem. Clearly much remains to be learned about the influence of problem characteristics on problem solving by analogy. In addition to investigating the effects of problem type, we need to learn more about the ways in which the use of analogies may interact with other strategies (e.g., means-ends analysis) used in problem solving.

Noticing and Accessing Potential Analogies

A number of important questions for future research involve the closely related issues of the spontaneous noticing of analogies, and the accessing of potential analogies stored in memory. The results of Experiments IV and V suggest that one of the major blocks to successful use of an analogy may be failure to spontaneously notice its pertinence to the target problem. When subjects were not told to try to use the prior stories to help solve the radiation problem, only a minority succeeded in generating the analogous solution. This decline in transfer performance cannot be attributed to faulty encoding of the story analogy, since most subjects were able to produce the analogous solution once they were given a hint to apply the story. Also, the problem of spontaneous noticing was not limited to stories previously encoded into memory. In the Story Second condition of Experiment V, many subjects failed to notice the relevance of the story even though they had to read, memorize, and recall it *after* beginning to work on the target problem.

However, even when subjects are not given an explicit hint to use a story analogy to solve a problem, the analogy itself can be viewed as a hint about a possible solution to the target problem. Previous work on the general topic of hints in problem solving, most notably that of Maier (1930, 1931), has used hints that involve objects that are incorporated into the solution directly, rather than analogically. It is therefore difficult to compare the present results with earlier research on hints. For example, in one investigation of the use of hints in the "two-string" problem, the experimenter "accidentally" brushed against one string and set it in motion (Maier, 1931). This hint was very effective in eliciting the "pendulum" solution (attaching objects as weights and setting the strings in motion); yet subjects rarely reported being aware of using the hint, unless they gave the solution in separate stages, rather than all at once. This experiment is most comparable to the conditions in Experiments IV and V in which an analogy was made available to subjects in the guise of a recall task, but subjects were not told that the story analogy was a hint to help solve the radiation problem. In apparent contrast to Maier's results, most subjects in these conditions who generated the dispersion solution reported using the story to do so. Thus it seems that our subjects were usually aware of the usefulness of the prior story, although the precise time at which they noticed its relevance is unclear from the data (since these were post-hoc reports).

Why should subjects so often fail to notice the relevance of a story analogy to a target problem when a hint to use the story is not provided? One might argue that this result is not particularly surprising, since the

story was presented in a different experimental context (a story recall experiment). The difficulty of the recall context may be related to the problem of identifying the optimal level of abstraction for representing an analogy, as we discussed in the Introduction. A recall task, with its emphasis on memory for specific wording, may lead the person to represent the story at a level of macrostructure too detailed to maximize its analogical correspondence with the target problem. A hint to use the story may lead the person to derive a more abstract level of macrostructure, better suited for the problem-solving task.

But in any case, the issue of how analogies are noticed is a very general one. A potential analogy may often be encoded in a very different context from that in which the target problem appears. Indeed, the basic problem in using an analogy between remote domains is to connect two bodies of information from disparate semantic contexts. More generally, successful transfer of learning generally involves overcoming contextual barriers. This may not be easy; for example, it is all too common for a student to fail to notice the relevance of knowledge acquired in one class to a problem encountered in another.

The problem of how analogies are noticed is closely related to the issue of how analogies are accessed in memory. Noticing that information in memory is relevant to a target problem is part of the process of retrieving an analogy. These problems were side-stepped in Experiments I-III, since subjects received a hint to use the story analogies and were allowed to reread them at any time. The problem of memory access was greatest in Experiment IV, in which the relevant story analogy was memorized in the context of two irrelevant distractor stories. Subjects in this experiment seemed to have little difficulty in identifying the appropriate story in memory, and applying it to the target problem, as long as they were instructed to do so. However, subjects may have performed this task by simply testing each of the three stories to see if it suggested a solution to the target problem. Such a strategy would presumably be impractical in most everyday problem-solving situations, where virtually any piece of information in memory might potentially afford a useful analogy.

How might potential analogies be accessed in memory? Is the memory search process directed, and if so, how? At one extreme the problem solver may not actually search memory at all; rather, he or she may simply "stumble upon" an analogy. That is, after a piece of knowledge has for some reason become the focus of attention, the person may spontaneously notice its analogous relationship to a problem yet to be solved. It also seems plausible, however, that people may sometimes locate useful analogies in memory on the basis of a conscious search process. It may be possible to use a representation of the current problem as a retrieval cue for accessing analogous problems. Perhaps in some cases the person

first begins working on a problem and arrives at an abstract characterization of a potential solution, as we discussed in Experiment I. This solution representation might then be used to retrieve an analogous problem with that type of solution, which could then be used to help generate a more concrete solution to the target problem. The latter possibility is related to the solution-focusing strategy discussed in connection with Experiment I. A better understanding of how analogies are retrieved and noticed is clearly essential in order to effectively teach the use of analogies as a heuristic strategy for problem solving (Polya, 1957).

The Generality of the Mapping Process

The mapping process involved in the use of analogies may play a role in a variety of cognitive skills. Using an analogy involves mapping the representations of two (or perhaps more) instances onto one another. Similar processes may also be involved in *abstracting* the relational structure common to a set of particular instances. In the domain of problem solving, for example, a person who encounters several analogies to the radiation problem might eventually derive a schema for "dispersion-type" problems. This schema would presumably be structured much like a concrete instance of a dispersion problem (cf. Figure 1), except that the predicates and arguments would be more abstract. A person equipped with such a general schema could then solve new dispersion-type problems by mapping them directly onto it. These observations suggest that similar mapping processes may be involved in three distinct but interrelated activities: (1) comparing one instance to another; (2) deriving a schema for a class of instances; and (3) comparing an instance to a general schema.

Note that the above description of the role of mapping potentially applies not just to problem solving, but to a wide range of cognitive skills requiring concept learning and classification of instances. Such skills are involved in tasks that vary a great deal in terms of both complexity and cognitive domain. For example, the mapping of correspondences between relational structures is involved in the use of schemata for story understanding (Rumelhart, 1975), frames for scene perception (Minsky, 1975), and scripts for understanding of social behavior (Abelson, 1975). Such structures all serve to describe our ability to deal with novel instances of familiar situations. Theories in each domain must explain how abstract structures can be derived from a set of instances, and how instances can be related to each other and to abstract structures.

If similar mapping processes are involved in analogical problem solving and other cognitive skills, then the study of the use of analogies to solve problems has implications that extend to other domains. We mentioned at the beginning of this paper that an analogy may often serve as a model to guide the development of a new theory. In a similar fashion a theory of

analogical problem solving might serve as a useful model in developing theories in other areas of cognition.

APPENDIX I

Story Analogies Used in Experiment I

First Paragraph (All Stories)

A small country fell under the iron rule of a dictator. The dictator ruled the country from a strong fortress. The fortress was situated in the middle of the country, surrounded by farms and villages. Many roads radiated outward from the fortress like spokes on a wheel. A great general arose who raised a large army at the border and vowed to capture the fortress and free the country of the dictator. The general knew that if his entire army could attack the fortress at once it could be captured. His troops were poised at the head of one of the roads leading to the fortress, ready to attack. However, a spy brought the general a disturbing report. The ruthless dictator had planted mines on each of the roads. The mines were set so that small bodies of men could pass over them safely, since the dictator needed to be able to move troops and workers to and from the fortress. However, any large force would detonate the mines. Not only would this blow up the road and render it impassable, but the dictator would then destroy many villages in retaliation. A full-scale direct attack on the fortress therefore appeared impossible.

Second Paragraph, Attack-Dispersion Story (Version 1)

The general, however, was undaunted. He divided his army up into small groups and dispatched each group to the head of a different road. When all was ready he gave the signal, and each group charged down a different road. All of the small groups passed safely over the mines, and the army then attacked the fortress in full strength. In this way, the general was able to capture the fortress and overthrow the dictator.

Second Paragraph, Open Supply Route Story

The general, however, was undaunted. He knew that one major thoroughfare leading to the fortress was always kept open as a supply route. He led his army to the head of the supply route. When all was ready he gave the signal, and the entire army charged down the open route. The army avoided the mines and attacked the fortress in full strength. In this way, the general was able to capture the fortress and overthrow the dictator.

Second Paragraph, Tunnel Story

The general, however, was undaunted. He and his men dug an underground tunnel beneath the mines following the route of the road to the

fortress. When the tunnel was dug, the men crawled through it until they arrived safely at the foot of the fortress. Here they all gathered together and attacked the fortress in full strength. In this way, the general was able to capture the fortress and overthrow the dictator.

APPENDIX II

Story Analogies Used in Experiment II

Attack-Dispersion Story (Version 2)

A small country was controlled by a dictator. The dictator ruled the country from a strong fortress. The fortress was situated in the middle of the country, surrounded by farms and villages. Many roads radiated outward from the fortress like spokes on a wheel. A general arose who raised a large army and vowed to capture the fortress and free the country of the dictator. The general knew that if his entire army could attack the fortress at once it could be captured. The general's troops were gathered at the head of one of the roads leading to the fortress, ready to attack. However, a spy brought the general a disturbing report. The ruthless dictator had planted mines on each of the roads. The mines were set so that small bodies of men could pass over them safely, since the dictator needed to be able to move troops and workers to and from the fortress. However, any large force would detonate the mines. Not only would this blow up the road and render it impassable, but the dictator would then destroy many villages in retaliation. It therefore seemed impossible to mount a full-scale direct attack on the fortress.

The general, however, knew just what to do. He divided his army up into small groups and dispatched each group to the head of a different road. When all was ready he gave the signal, and each group marched down a different road. Each group continued down its road to the fortress, so that the entire army finally arrived together at the fortress at the same time. In this way, the general was able to capture the fortress, and thus overthrow the dictator.

Parade-Dispersion Story

A small country was controlled by a dictator. The dictator ruled the country from a strong fortress. The fortress was situated in the middle of the country, surrounded by farms and villages. Many roads radiated outward from the fortress like spokes on a wheel. To celebrate the anniversary of his rise to power, the dictator ordered his general to conduct a full-scale military parade. On the morning of the anniversary, the general's troops were gathered at the head of one of the roads leading to the fortress, ready to march. However, a lieutenant brought the general a disturbing report. The dictator was demanding that this parade had to be more impressive than any previous parade. He wanted his army to be

seen and heard at the same time in every region of the country. Furthermore, the dictator was threatening that if the parade was not sufficiently impressive he was going to strip the general of his medals and reduce him to the rank of private. But it seemed impossible to have a parade that could be seen throughout the whole country.

The general, however, knew just what to do. He divided his army up into small groups and dispatched each group to the head of a different road. When all was ready he gave the signal, and each group marched down a different road. Each group continued down its road to the fortress, so that the entire army finally arrived together at the fortress at the same time. In this way, the general was able to have the parade seen and heard through the entire country at once, and thus please the dictator.

APPENDIX III

Story Analogy Used in Experiments IV and V

Attack-Dispersion Story (Version 3)

(1) A small country was ruled (2) [from a strong fortress] by a king. (3) The fortress was situated in the middle of the country (4) surrounded by farms and villages. (5) Many roads radiated outward from the fortress (6) like spokes on a wheel. (7) A rebel general vowed (8) to capture the fortress. (9) The general knew that (10) an attack (11) [by his entire army] would capture the fortress. (12) He gathered his army (13) at the head of one of the roads. (14) However, the general learned that (15) the king had planted mines (16) on each of the roads. (17) The mines were set (18) so that (19) [small] bodies of men could pass over them safely, (20) since the king needed (21) to move his troops and workers to and from the fortress. (22) However, any (23) [large] force would detonate the mines. (24) Not only would this blow up the road (25) and render it impassable, (26) but it would also destroy many neighboring villages. (27) It therefore seemed impossible (28) to mount a (29) [full-scale direct] attack on the fortress. (30) The general, however, knew just what to do. (31) He divided his army up into small groups (32) and dispatched each group (33) to the head of a different road. (34) When all was ready (35) he gave the signal (36) and each group marched down a different road. (37) Each group continued down its road to the fortress (38) so that the (39) [entire] army finally arrived (40) [together] at the fortress at the same time. (41) The fortress fell (42) and the king was forced (43) to flee into exile.

APPENDIX IV

Distractor Stories Used in Experiment IV

The Wine Merchants

One day a rich man found that his wine cellar was empty. So he sent out messengers to announce a generous offer. The first person to bring the

rich man a barrel of wine would be given a brick of solid gold. However, the offer would expire at sundown.

Two wine merchants heard the news. Each had a horse-drawn cart loaded with large barrels of wine. They both set out for the duke's palace at once. An hour before sundown they came to a place where the bridge had been washed out by a raging river. The first merchant drove his horses and cart into the flood in a desperate attempt to reach the other side. But the horses were already exhausted and could not fight the current. The cart overturned, and the horses, wine, and driver were washed away.

The second merchant tried a different tactic. He poured the wine out of all but one of his barrels, and lashed them together to form a raft; then he loaded the one full barrel, a horse, and himself on top. He set the raft adrift and floated downstream. In a few minutes the raft came to rest on the shore in front of the town where the rich man lived. The merchant disembarked, loaded the wine barrel on the horse, and led it to the rich man's house. He arrived just as the sun was setting, and collected the gold brick as a reward for his efforts.

The Identical Twins

Once there were identical twins who were continually playing pranks on their family, friends, and teachers. The annual school picnic was always a big event for the twins. There were races and other athletic events in which the twins won lots of prizes. One year a new student arrived who was a star runner. The twins wanted to win the main event: the 2-mile race through the woods behind the school. So they secretly devised a plan which would enable them to outdo the newcomer.

The day of the race arrived. Each runner was to pick his own path through the woods to a clearing, where a teacher stood posted to determine the winner. One twin entered the race, while the other excused himself on the grounds that he had hurt his leg in an earlier broadjumping event. The race began and the students rushed into the woods. The twin rushed into the woods and waited until the others had passed out of sight. Then he went back to the school using a path hidden from the picnic area. Shortly after, the other twin, who had been hiding behind a rock near the finish line of the race, burst out and ran into the clearing ahead of the other runners. The teacher named him the winner and marveled at the speed of his running. Next year the twins switched places and thereafter maintained their status on this event.

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