Journal of Experimental Psychology:
Human Perception and Performance
2014, Vol. 40, No. 1, 71–82

0096-1523/14/$12.00 DOI: 10.1037/a0032916

Magnitude Comparison With Different Types of Rational Numbers

Melissa DeWolf
University of California, Los Angeles

Margaret A. Grounds and Miriam Bassok
University of Washington

Keith J. Holyoak
University of California, Los Angeles

An important issue in understanding mathematical cognition involves the similarities and differences between the magnitude representations associated with various types of rational numbers. For single-digit integers, evidence indicates that magnitudes are represented as analog values on a mental number line, such that magnitude comparisons are made more quickly and accurately as the numerical distance between numbers increases (the distance effect). Evidence concerning a distance effect for compositional numbers (e.g., multidigit whole numbers, fractions and decimals) is mixed. We compared the patterns of response times and errors for college students in magnitude comparison tasks across closely matched sets of rational numbers (e.g., 22/37, 0.595, 595). In Experiment 1, a distance effect was found for both fractions and decimals, but response times were dramatically slower for fractions than for decimals. Experiments 2 and 3 compared performance across fractions, decimals, and 3-digit integers. Response patterns for decimals and integers were extremely similar but, as in Experiment 1, magnitude comparisons based on fractions were dramatically slower, even when the decimals varied in precision (i.e., number of place digits) and could not be compared in the same way as multidigit integers (Experiment 3). Our findings indicate that comparisons of all three types of numbers exhibit a distance effect, but that processing often involves strategic focus on components of numbers. Fractions impose an especially high processing burden due to their bipartite (a/b) structure. In contrast to the other number types, the magnitude values associated with fractions appear to be less precise, and more dependent on explicit calculation.

Keywords: magnitude representation, distance effect, rational numbers, mental number line, fraction comparisons

A central issue in understanding mathematical cognition involves determining the nature of mental representations of numerical magnitudes. A ubiquitous phenomenon found for stimuli that can be ordered along a perceptual or symbolic dimension is that a magnitude comparison (e.g., choosing the numerically larger of two digits) is made more quickly and accurately the greater the magnitude difference between the stimuli (Holyoak, 1978; Moyer & Landauer, 1967). Such *distance* effects have also been observed with children (e.g., Barth, Mont, Lipton, & Spelke, 2005; Brannon, 2002; Brannon & Van De Walle, 2001). For digit comparisons, the distance effect typically follows a logarithmic function, such that comparisons are more difficult for larger magnitude values when the numerical difference is held constant (Moyer & Landauer, 1967). The distance effect has been interpreted as evidence that numerical and other magnitudes are coded in an analog form akin to a mental number line (Dehaene & Changeux, 1993; Gallistel, 1993).

Research on the distance effect has been extended to multidigit integers, focusing on the extent to which adults use componential or holistic strategies to process the integers. The earliest such study (Hinrichs, Yurko, & Hu, 1981) found evidence of holistic processing, but more recent work using closely matched pairs of comparisons has revealed evidence for componential processing of whole numbers. For example, Nuerk, Weger, and Willmes (2001) found that when the numbers in the tens place and unit digit were incompatible (e.g., 47 vs. 62), comparisons were slower and more error-prone than comparisons in which the tens and unit digits were compatible (e.g., 52 vs. 67). Further, Verguts and De Moor (2005) found that adults show a distance effect when comparing double-digit integers of the same decade; however, no distance effect was apparent for pairs that were the same distance apart but in different decades (i.e., a distance effect for pairs like 51 vs. 58, but not for pairs

This article was published Online First June 10, 2013.

Melissa DeWolf, Department of Psychology, University of California, Los Angeles, Los Angeles, California; Margaret A. Grounds and Miriam Bassok, Department of Psychology, University of Washington, Seattle, Washington; Keith J. Holyoak, Department of Psychology, University of California, Los Angeles, Los Angeles, California.

Preparation of this article was supported by Grant R305C080015 from the Institute of Education Sciences. The results of Experiment 1 were presented at the 52nd Annual Meeting of the Psychonomic Society in Seattle, Washington, November 2011. We thank Airom Bleicher for help in collecting and analyzing data, and Jim Stigler, Belinda Thompson, Monica Rapp, and Jessica Walker for helpful discussions.

Correspondence concerning this article should be addressed to Melissa DeWolf, Department of Psychology, University of California, Los Angeles, 405 Hilgard Ave., Los Angeles, CA 90095-1563. E-mail: mdewolf@ucla.edu

like 45 vs. 52; see also Dehaene, Dupoux, & Mehler, 1990). In addition, Ganor-Stern, Tzelgov, and Ellenbogen (2007) found evidence for componential processing of double-digit numbers based on a more implicit measure (the size congruity effect), which was influenced by the compatibility of the units and tens digits, but not the magnitude of the number as a whole. On balance, the weight of evidence suggests that multidigit integers are processed componentially rather than holistically. Although distance effects have been observed for comparisons of multidigit integers, the effects often depend on comparisons of component digits, rather than on integrated magnitudes associated with the entire numbers.

Magnitude Representations for Fractions and Decimals

In addition to comparing the magnitudes of single digits and multidigit whole numbers, educated children and adults can also compare other types of rational numbers, notably fractions and decimals. Given the evidence reviewed indicating that comparisons of multidigit integers are often componential in nature, it is natural to hypothesize that similar componential processing may be involved with other rational numbers. Fractions and decimals are introduced in school later than integers, and a great deal of evidence indicates that children have difficulty understanding these number types (Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004). What is more controversial is whether adults with high education levels (e.g., college students) eventually acquire magnitude representations for other rational numbers that are basically similar to their representations of integer magnitudes.

Many researchers have examined the extent to which the mental representation of fractions is the same as or different from representations of integers (e.g., Bonato, Fabbri, Umiltà, & Zorzi, 2007; Schneider & Siegler, 2010; Siegler, Thompson, & Schneider, 2011), but the extent to which the distance effect extends to magnitude comparisons based on fractions is still unclear. Bonato et al. (2007) found no evidence for a distance effect related to the relative magnitudes of the fractions in the comparisons. These investigators argued that adults process fractions in a piecemeal manner, dividing up the parts of the fractions (the numerator and denominator), and using heuristics based on these separate parts to determine the relative sizes of the fractions. However, Schneider and Siegler (2010) argued that a distance effect can be obtained for fractions, but only if the stimuli require processing of the integrated magnitude of the fraction, rather than just its parts. For example, Bonato et al. (2007) examined unit fractions in the form $1/n$, which do not require adults to compare the integrated magnitudes of the fractions. Instead, people could use a simple heuristic: Fractions that have larger denominators are smaller overall. Thus, they could just compare the sizes of the denominators to determine which fraction was larger. When Schneider and Siegler (2010) tested comparisons based on fractions for which this heuristic could not be used, they found a pattern consistent with the hypothesis that adults do in fact use the integrated magnitude of the fraction to perform the comparison.

Formal and Conceptual Distinctions Among Types of Rational Numbers

Though adults may be capable of generating integrated magnitudes for fractions, there is also evidence that fractions pose extra processing burdens even for adults (Givvin, Stigler, & Thompson, 2011; Siegler et al., 2011; Stigler, Givvin, & Thompson, 2010; see Richland, Stigler, & Holyoak, 2012). There are some characteristic differences between fractions and integers that seem likely to impact the comparison process. One key difference is in their perceptual form: Fractions are not represented with a unitary symbol, but rather have a bipartite structure, being composed of a separate numerator and denominator. This formal difference between fractions and integers potentially appears to contribute to early misconceptions, and also to later use of fallible shortcut strategies for performing computations with fractions (Bonato et al., 2007).

Other key differences are more inherently conceptual: For example, unlike whole numbers, fractions do not have unique successors. This and related conceptual differences play a crucial role throughout development, especially when children attempt to integrate fractions into their already well-established understanding of whole numbers (Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004; see Siegler, Fazio, Bailey, & Zhou, 2013, for a review). For example, Vamvakoussi and Vosniadou (2004) found that Greek 12-, 14-, and 16-year-olds made several typical mistakes when completing tasks relating to the infinite number of rational numbers in any interval of a number line. The young adolescents seemed to improperly extend characteristics of integers to other rational numbers. Their misconceptions included the idea that fractions are discrete and have unique successors, as integers do, and that only fractions can lie between two fraction endpoints, whereas only whole numbers can lie between two whole-number endpoints.

In evaluating the possible impact of the formal and conceptual differences between fractions and whole numbers on processing difficulty and magnitude representation, a comparison with decimals would appear to be potentially illuminating. Like fractions, decimals denote rational numbers that lack a unique successor. But in terms of their form, decimals are not explicitly decomposed into two major parts (numerator and denominator), as are fractions, but rather are more similar to integers. Decimals are typically introduced in the curriculum after fractions (and whole numbers). Difficulties have been observed when children attempt to integrate knowledge about decimals with their knowledge of whole numbers. Vamvakoussi and Vosniadou (2004) found that children inappropriately extend properties of whole numbers to decimals. Rittle-Johnson, Siegler, and Alibali (2001) found that children improperly view the number of digits in a decimal as an indication of its magnitude (e.g., a majority of fifth and sixth graders claimed that .274 is larger than .83). Such misconceptions suggest that the conceptual distinctions that separate fractions and decimals, on the one hand, from whole numbers, on the other, may be especially significant in making the former types of numbers harder to grasp.

However, there is also evidence suggesting that children can integrate decimals into their mathematical knowledge more easily than they can fractions. Iuculano and Butterworth (2011) used a number line task to assess the accuracy of adults and children in locating integers, decimals, and fractions on a number line. They found that performance was quite similar for decimals and integers. In contrast, adults and children (aged 10 years) were less accurate when using fractions, compared with decimals and integers, in placing a mark on the number line at a certain value, and were even less accurate when asked to generate a given value using fractions when shown a mark on a number line.

Such findings suggest that, at least for well-educated adults, processing decimals may be less computationally demanding than processing fractions, and that processing decimals may be more similar to processing integers. Surprisingly, however, no study of magnitude comparisons has directly compared patterns of performance with fractions and decimals. Indeed, only one study has been reported that examined magnitude comparisons with decimals (Cohen, 2010). This study found that, in addition to the typical distance effect observed with comparisons of integers, magnitude comparisons with decimals are mediated by a decimal-specific strategy.

Goals of the Present Study

In order to examine the similarities and differences among magnitude representations for distinct types of rational numbers, we investigated how adults complete magnitude comparisons across three types of rational numbers: fractions, decimals, and three-digit integers. By focusing on adult performance, we aimed to illuminate the end state of the acquisition of magnitude codes for the various number types. For each type of number, we sought to determine whether or not a distance effect is obtained, as this effect is generally considered to be a key "signature" of a magnitude representation based on an internal number line. In addition, we sought to determine whether comparisons are based on integrated magnitude representations or on componential processing of subparts of the numbers. For this reason, we focused on number types with compositional structure.

In Experiment 1, we performed the first direct comparison of magnitude comparisons based on fractions and decimals. One hypothesis, based on the fact that fractions are introduced in school prior to decimals, is that fractions may be overall easier to process. A second hypothesis is based on the argument that although fractions and decimals are different in form, they share conceptual properties of noninteger rational numbers. Accordingly, if their conceptual similarity is critical, fractions and decimals may be equally easy (or hard) to process. A third hypothesis stems from the perceptual differences in format between fractions and decimals. Fractions have explicit parts, whereas decimals are more unitized. If this difference is critical, then decimals will be overall easier. In Experiments 2 and 3, we added comparisons based on three-digit integers, enabling us to directly compare processing of decimals and fractions with processing of compositional integers. One possibility is that magnitudes are more difficult to assess for fractions and decimals than for integers, due to the more complex conceptual structure of both of these types of rational numbers. However, if fractions impose special processing burdens because of their bipartite format (e.g., requiring finding a common denominator, or perhaps even translation into decimal form), then fractions may be more difficult to compare than either of the other number types.

Experiment 1

Method

Participants. Participants were 69 undergraduates from the University of Washington (mean age = 19.6 years; 39 females).

Design and materials. The design and stimuli of Experiment 1 were modeled closely on the study reported by Schneider and Siegler (2010), with an additional within-subjects condition of number type (decimals as well as fractions). This task formed one component of a larger study involving several additional mathematical reasoning tasks (not reported here). Participants completed a series of magnitude comparisons organized as separate blocks of fractions and decimals. The order of the fraction and decimal blocks was counterbalanced across subjects. The fraction block began with practice comparisons (simpler problems in which both numerator and denominator consisted of a single digit), followed by the target fraction comparisons. Similarly, the decimal block began with practice decimal comparisons (equivalent in magnitudes to the practice fractions), followed by the target decimal comparisons. Order of problems was randomized within each block. The decimals were the magnitude equivalents of the fractions (see Table 1 for a complete list of fractions and decimals used in the comparisons). The decimals were all rounded to three digits to make the decimals consistent in length regardless of their fraction equivalent. In addition, using three digits helped to standardize the number of digits participants had to process across number type (the fractions had between two and four digits). All of the comparisons were done against the reference value of 3/5 for fractions or 0.600 for decimals.

Table 1
Fractions and Decimals Used As Targets in Experiment 1, Paired With Their Alphabetical Code Used in Figures 1 and 2

Code	Fraction	Decimal
a	20/97	0.206
b	1/4	0.250
c	26/89	0.292
d	30/91	0.330
e	28/71	0.394
f	31/72	0.431
g	32/69	0.464
h	25/49	0.510
i	23/44	0.523
j	33/62	0.532
k	5/9	0.556
l	29/51	0.569
m	24/41	0.585
n	22/37	0.595
o	27/43	0.628
p	37/58	0.638
q	35/54	0.648
r	36/53	0.679
s	38/55	0.691
t	40/57	0.702
u	41/56	0.732
v	47/59	0.797
w	43/48	0.896
x	49/52	0.942
y	46/47	0.979

Procedure. Participants were presented with 12 practice trials and 25 test trials for both fractions and their decimal equivalents. Within each block, the 25 target trials were repeated five times (in a new random order each time). After the five random iterations of the trial lists, the participants would move on to the next number type. On each trial, the target fraction or decimal was displayed at the center of the screen. Half of the participants were instructed to hit the *h* key if the target fraction (or decimal) was larger than the reference value of 3/5 (or 0.600) and to hit the *g* key if the number was smaller than 3/5 (or 0.600); the other half were given the reverse key assignments. A reminder was written at the bottom of each screen as to which key to hit if the target was larger or smaller.

Participants were instructed to respond as quickly and accurately as possible. They were given a maximum of 5 s to make the comparison. If they had not made a selection by that time, the screen moved on to the next trial. Participants were allowed to rest briefly between each block of trials.

Results and Discussion

Mean error rate was calculated for each target item for each participant and compared across number type (fraction vs. decimal). As there was no effect of presentation order for number types (fractions: $t[68] = .75$, $p > .05$; decimals: $t[68] = .81$, $p > .05$), all results are reported after collapsing across this counterbalancing condition. The pattern of error rates for different target magnitudes of each number type is shown in Figure 1. A distance effect is clearly present for fractions, as error rates increased as target values approached the reference value of 3/5. Decimals, by contrast, yielded perfect accuracy for all target values. Collapsing across all targets, the mean error rate for fraction comparisons was 5.6% ($SD = 6.4$), versus 0% for decimal comparisons, a difference that was highly reliable by a paired-samples *t* test, $t(68) = 7.25$, $p < .001$. Thus, participants were substantially less accurate for fraction comparisons than for decimal comparisons.

Mean response time (reaction time [RT]) to make a correct judgment was also calculated for each participant and compared across number type (fraction vs. decimal). RTs for trials that were answered incorrectly, or exceeded the response window, were not included in these analyses. The pattern of RT values for different target magnitudes of each number type is shown in Figure 2. The fraction comparison is clearly slower than the matched decimal comparison for every matched target value, and the fractions show a more dramatic increase in RT around the reference value. Collapsing across all targets, mean RT for fractions (1.34 s; $SD = 0.47$ s) was significantly slower (in fact, twice as long) as that for decimals (.65 s; $SD = 0.12$; $t[68] = 12.60$, $p < .001$).

In order to assess the functional form of the distance effect for response times, we considered three models that have often been used in previous studies of numerical comparisons: linear distance between the target and reference value (Bonato et al., 2007); logarithm of the linear distance (i.e., log (|target—reference|), which we will abbreviate as "log Dist" (Dehaene et al., 1990; Hinrichs et al., 1981; Schneider & Siegler, 2010); and the Welford function, log (Larger number/[Larger number − Smaller number]) (Moyer & Landauer, 1967; Hinrichs et al., 1981). Of these, the linear distance was the least successful, as the form of the RT function in Figure 2 clearly shows a negatively accelerated relation between RT and distance from the reference value. For the problems used in Experiment 1 (and also Experiments 2 and 3), the logarithm of linear distance was extremely highly correlated with the Welford function, $r = -0.99$, $p < .001$; thus, as a practical matter, these two functions were difficult to distinguish for our study. The Welford function predicts a subtle asymmetry (higher RTs for comparisons of target values above vs. below the reference value, when matched for linear distance), which is not evident in

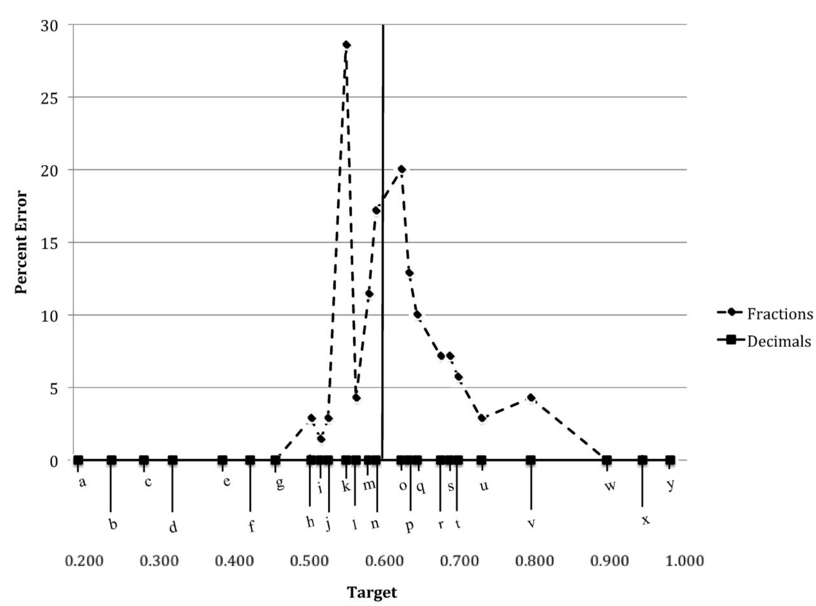

Figure 1. Distribution of percent errors across target magnitudes for fractions and decimals (Experiment 1). The solid vertical line marks the reference value. See Table 1 for identities of targets indicated by code on x-axis.

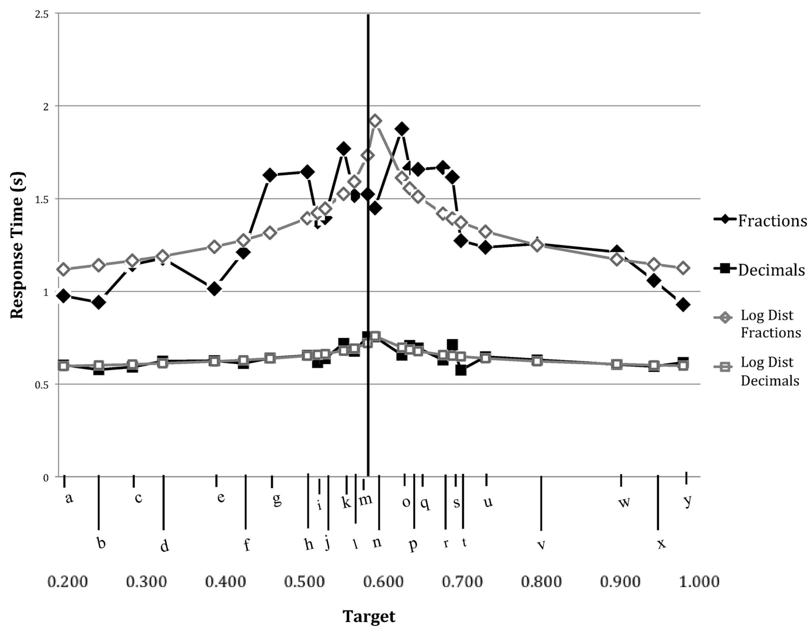

Figure 2. Distribution of mean response times across target magnitudes for fractions and decimals (Experiment 1). The solid vertical line marks the reference value. See Table 1 for identities of targets indicated by code on x-axis.

our data (see Figure 2). Accordingly, we selected the logarithmic distance model (log Dist) for use in the analyses reported here. Because accuracy was at ceiling for decimals, we focused on distance effects based on the RT measure (see Figure 2). Log Dist accounted for 51% of the variance for fractions and 67% of the variance for decimals. The regression coefficients were significant for both fractions ($\beta = -0.73$, $t[23] = 5.14$, $p < .001$), and decimals ($\beta = -0.82$, t[23[= 6.82, $p < .001$). Thus, for both the fractions and decimals, the pattern of response times follows a logarithmic function consistent with the distance effect.

An important question is whether fractions are simply slower to compare than decimals by a constant increment of time, or whether the slope of the distance function differs between the two number types. If the extra difficulty of fractions is solely due to the time required to calculate a magnitude, then the slope of the distance function would not be expected to differ across the two number types. However, if the magnitude representation derived for fractions is less precise than that for decimals, then formal models of magnitude comparison (e.g., Chen, Lu & Holyoak, 2013; Marks, 1972) would predict that the distance effect will be more pronounced for fractions. A linear regression revealed that the logarithmic distance between the target and the reference value significantly predicted the *difference* in RT between fraction and decimal comparisons for the same values, ($\beta = -0.65$; $t[23] = 4.10$, $p < .001$; adj. $R^2 = .40$). The unstandardized coefficients yielded a value of -0.34 for the slope of the line, indicating that the difference in RT (fraction—decimal) decreases by about 0.34 s for every unit increase in the log distance from the reference. Thus, although both fractions and decimals show a distance effect, the impact of distance on RT is substantially greater for comparisons of fractions than of decimals (i.e., fractions are especially hard relative to decimals when the distance to the reference value is small.) This finding indicates that the extra difficulty of fractions is not solely in initial generation of a magnitude code but also in the comparison process itself, perhaps due to reduced precision of magnitude values for fractions (Chen et al., 2013). That is, fractions may elicit less precise magnitude values than do decimals, perhaps due to greater reliance on approximate estimates when interpreting fractions.

In the case of decimals, we also wished to determine whether people used the entire decimal in making comparisons or focused exclusively on the first significant digit (i.e., the tenths value, indicated by the first digit after the decimal point). In order to address this question, we performed a multiple regression analysis similar to the analyses reported previously, except that log Dist was computed using the value of the target after truncation to a single digit (its tenth value). If decisions were based solely on the first digit, than adding log Dist calculated using the full three-digit decimal would not explain additional reliable variance beyond that accounted for using log Dist after truncation. For five of the digit targets, truncation yielded a value of .6, and hence a distance of 0, for which the logarithm is undefined; these targets were excluded from the regression analysis. The first predictor based on log Dist after truncation predicted 24% of the variance in decimal RTs ($\beta = -0.52$, $t[22] = 2.88$, $p = .009$). However, adding the second predictor, log Dist based on the full three-digit decimal, significantly increased the explained variance to 74%, $F(1, 22) = 43.84$, $p < .001$. This finding indicates that participants based decimal comparisons on more than just the initial digit. However, as we will see, the results of Experiments 2 and 3 indicate that people can strategically focus on the first digit. These later experiments did not involve repeated testing of the same numbers, as was done in Experiment 1. Thus, it may be that repeated testing encourages more holistic processing of decimals.

Given that RTs and error rates were far higher overall for fractions than decimals, we sought to determine whether the difficulty of fractions was reduced for those that were relatively simple in form. We identified 1/4 and 5/9 as relatively "simple" fractions because they have single-digit numerators and denominators, whereas all the other target fractions have double digits. Comparing the RTs of 1/4 with 26/89 and 20/97 (the two fractions closest in magnitude to 1/4), we found that 1/4 is significantly faster than 26/89 (.94 s vs. 1.14 s; $t[136] = 2.72$, $p = .007$), but not 20/97 (.94 s vs. .98 s; $t[136] = 0.50$, $p = .61$). This pattern is consistent with the distance effect, as 1/4 is further from the reference value than is 26/89. Hence, this finding does not suggest a special advantage for the simple fraction. Comparing 5/9 with its two closest values, 33/62 and 29/51, 5/9 was actually significantly slower than 33/62 (1.8 s vs. 1.4 s; $t[114] = 2.37$, $p = .02$) and not significantly different from 29/51 (1.8 s vs. 1.5 s; $t[113] = 1.49$, $p = .14$). Thus, we did not find any compelling evidence that the two "simple" fractions were evaluated any more quickly than would be predicted by distance alone. It is possible that if any special strategies are potentially applicable for simpler fractions, these were not used in the present experiment, in which the great majority of the target fractions were of the more complex format (two-digit numerators and denominators).

Overall, the findings of Experiment 1 demonstrate that both fractions and magnitude-matched decimals show reliable distance effects: Comparisons of both types of rational numbers are faster (and for fractions, more accurate) as the numerical distance between the target and reference values increases. But even though fractions and decimals are conceptually very similar, there was a clear decimal advantage, as comparisons of fractions proved to be much slower and more error-prone. In addition, the distance effect was more dramatic for fractions than for the corresponding decimals, with the impact of distance accelerating more quickly for fractions. These findings support the hypothesis that the formal structure of numerical representations influences both overall processing difficulty and the precision of magnitude representations. Decimals appear to benefit from their greater formal similarity to integers. Experiment 2 was performed to more directly compare performance with integers to that with fractions and decimals.

Experiment 2

Experiment 2 was designed to replicate and extend the findings of Experiment 1 by adding an additional number type, three-digit integers, in addition to a number of other methodological changes. By including all three number types in a single experiment, it is possible to directly compare the pattern of performance across all these types of rational numbers. In light of the results of Experiment 1, which revealed much greater difficulty in processing fractions than decimals, we were particularly interested in determining whether or not comparisons of decimals are, in turn, more difficult than comparisons of corresponding integers.

Method

Participants. Participants were 95 undergraduates from the University of California, Los Angeles (mean age = 21 years; 72 females), who received course credit.

Design and materials. Three different types of numbers were used for magnitude comparisons: integers, decimals, and fractions. Number type was manipulated between participants so that each participant only received one type of number (in contrast to the within-subjects design used in Experiment 1), thereby avoiding any possible carryover effects of strategies that might be evoked by a particular number type. There were 34 participants in the fraction condition, 30 in the decimal condition, and 31 in the integer condition. As in Experiment 1, dependent measures were percent error and response time.

Table 2 lists the complete set of fractions, decimals, and integers used. Because the decimal advantage in Experiment 1 might be attributed, in part, to the relative simplicity of the reference value for decimals (0.600), in Experiment 2, we instead set the reference value for decimals at 0.613, thus making the reference value more complex. The symmetric distribution of target numbers around the reference value was maintained, such that half of the decimal targets were greater than the reference value. As the integers were simply 1,000 times the value of the decimals, 613 was used as the reference for the integers.

Procedure. Participants received either fractions, decimals, or integers to compare. The 30 trials were each presented a single time in random order. Participants were told to complete the comparison as quickly and accurately as possible. Unlike Experiment 1, no fixed deadline was imposed on time to reach a decision. The given number for the particular trial appeared in the center of the screen; participants had to select either the *a* key or the *l* key to indicate whether the number was larger or smaller

Table 2
Fractions, Decimals, and Integers Used As Targets in Experiment 2, Paired With Their Alphabetical Code Used in Figures 3 and 4

Code	Fraction	Decimal	Integer
a	20/97	0.206	206
b	1/4	0.250	250
c	26/89	0.292	292
d	30/91	0.330	330
e	28/71	0.394	394
f	31/72	0.431	431
g	32/69	0.464	464
h	1/2	0.500	500
i	25/49	0.510	510
j	23/44	0.523	523
k	33/62	0.532	532
l	5/9	0.556	556
m	29/51	0.569	569
n	24/41	0.585	585
o	22/37	0.595	595
p	27/43	0.628	628
q	37/58	0.638	638
r	35/54	0.648	648
s	2/3	0.667	667
t	36/53	0.679	679
u	38/55	0.691	691
v	40/57	0.701	701
w	5/7	0.714	714
x	41/56	0.732	732
y	39/50	0.780	780
z	47/59	0.797	797
aa	7/8	0.875	875
ab	43/48	0.896	896
ac	49/52	0.942	942
ad	46/47	0.979	979

than the reference value. As in Experiment 1, there was a reminder on the right side of the screen that said "larger than 3/5" (or the reference value appropriate to the number type) and "smaller than 3/5" on the left side.

Results and Discussion

Figure 3 presents the mean error rates for each target, for each of the three number types. Error rates were much higher for fractions ($M = 18.73$; $SD = 12.17$) than for decimals ($M = 3.22$; $SD = 4.06$) or integers ($M = 3.76$; $SD = 3.31$). A one-way between-subjects ANOVA showed that differences among number types were highly reliable, $F(2, 92) = 40.94$, $p < .001$. Planned contrasts revealed a significant difference in error rates between the fraction number type and the decimal and integer number types, $t(92) = 9.05$, $p < .001$, but no significant difference between decimal and integer number types, $t(92) = 0.27$, $p = .79$.

The pattern of response times across all three number types is depicted in Figure 4 (left panel). RTs were far faster for decimals and integers than for fractions. For clarity, the pattern for decimals and integers is also depicted in Figure 4 (right panel) after rescaling the y-axis to fit the faster time scale of these two number types. Collapsing over all targets, RTs were slower for comparisons of fractions ($M = 3.3$; $SD = 1.75$) than of decimals ($M = 0.86$, $SD = 0.21$) or integers ($M = 0.86$; $SD = 0.28$). A one-way ANOVA yielded reliable overall differences among number types, $F(2, 92) = 58.60$, $p < .001$. Planned contrasts revealed that comparisons of fractions were reliably slower than those between decimals or integers, $t(92) = 10.83$, $p < .001$, whereas RTs did not differ significantly between the latter number types, $t(92) = 0.05$, $p = .96$.

As in Experiment 1, error rates for comparisons of fractions showed a clear distance effect (see Figure 3), whereas errors for the other two number types were uniformly low. For the response time data (see Figure 4), we again performed regression analyses based on the logarithmic distance measure. Log Dist accounted for a significance amount of variance for all three number types: 76% for fractions, 30% for decimals, and 54% for integers (for fractions, $\beta = -0.883$, $t[28] = 9.97$, $p < .001$; for decimals, $\beta = -0.57$, $t[28] = 3.64$, $p = .001$; for integers, $\beta = -.751$, $t[28] = 6.01$, $p < .001$). From inspection of the data shown in Figure 4 (right), it appears that the relatively low regression fit for decimal comparisons reflects noise in the RT pattern. Note that the corresponding fit for decimals was considerably higher in Experiment 1 (67% of variance accounted for).

Intercorrelations between the RT patterns for the three number types revealed that although all three sets of RTs were related, decimals and integers had the strongest relation, $r(28) = 0.75$, $p < .001$, compared with fractions and integers, $r(28) = 0.48$, $p = .007$, or fractions and decimals, $r(28) = 0.36$, $p < .05$. As in Experiment 1, the distance effect was more pronounced for fractions than for the other number types. Linear regressions indicated that log Dist significantly predicted the *difference* in RT for fractions and decimals ($\beta = -0.73$, $t[28] = 5.71$, $p < .001$, adj. $R^2 = 0.52$), and for fractions and integers ($\beta = -0.73$, $t[28] = 5.62$, $p < .001$, adj. $R^2 = 0.51$), but not for the difference in RT for decimals and integers ($\beta = 0.23$, $t[28] = 1.22$, $p = .23$, adj. $R^2 = 0.02$).

As in Experiment 1, we performed a multiple regression analysis to assess whether participants made use of the entire three-digit decimal in making comparisons or just the tenth value (i.e., first digit). We again calculated log Dist based on

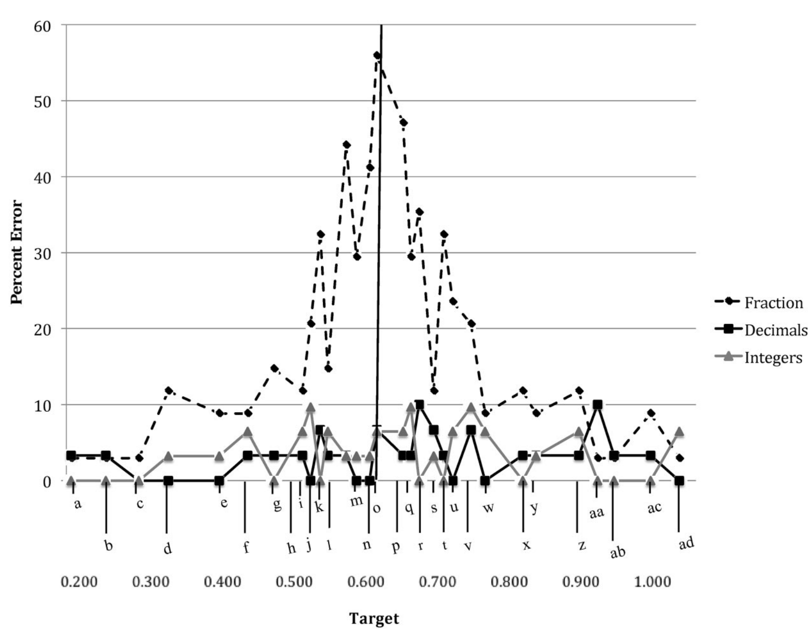

Figure 3. Distribution of percent errors across target magnitudes for fractions, decimals, and integers (Experiment 2). The solid vertical line marks the reference value. See Table 2 for identities of targets indicated by code on x-axis.

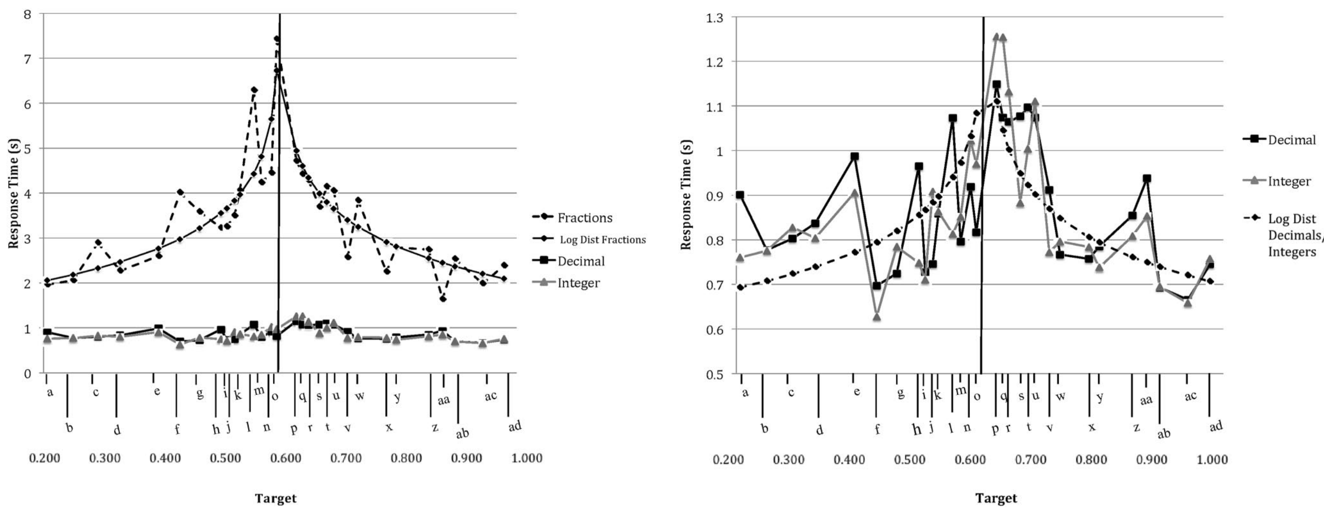

Figure 4. Left: Distribution of mean response times across target magnitudes for fractions, decimals, and integers (Experiment 2). Right: Same data, rescaled for decimals and integers only. The solid vertical line marks the reference value. See Table 2 for identities of targets indicated by code on x-axis.

truncating target decimals to their tenths place only (keeping the reference value as .613 so as to avoid the need to drop targets that truncate to .6). When log Dist based on truncation was used as the sole predictor, it accounted for 49% of the variance ($\beta = -0.71$, $t[28] = 5.40$, $p < .001$). In contrast to the similar analysis in Experiment 1, adding log Dist based on the full three-digit decimal did not significantly increase the variance accounted for (49%), $F(1, 27) = .21$, $p = .65$. As noted above, the RT pattern for decimals in Experiment 2 was relatively noisy (perhaps because the data were derived from a single trial for each participant, without extensive practice). The decimal RTs in Experiment 1, which are based on averaging over five trials, were considerably less variable. The greater variance of decimal RTs in Experiment 2 may have made RTs less sensitive to the detailed structure of the three-digit decimals.

We also conducted an analogous multiple regression on the integer RTs, using log Dist based on truncation to the first digit (hundreds place). This variable accounted for a significant amount of variance ($R^2 = .53$, $\beta = -0.74$, $t[28] = 5.84$, $p < .001$). However, adding log Dist based on the full three-digit integer yielded a reliable further increase in variance accounted for from (total $R^2 = 61\%$; $F[1, 27] = 6.56$, $p = .02$). Thus, we again found evidence that comparison of multidigit integers was based on more than just the first digit.

As in Experiment 1, we performed additional analyses to assess whether "simpler" fractions (those with one-digit numerators and denominators) were evaluated more quickly than would be predicted by distance to the reference value. We again found no evidence that this was the case. For example, RT for 1/4 was not significantly different from that for 20/97 (2.06 s vs. 1.96 s; $t[39] = 0.50$, $p = .62$) or 26/89 (2.06 s vs. 2.90 s; $t[35] = 0.03$, $p = .79$). Even 1/2, arguably the most common fraction of all, was not significantly faster than either 32/69 (3.23 s vs. 3.58 s; $t[57] = .53$; $p = .60$) or 25/49 (3.23 s vs. 3.25 s; $t[55] = 0.04$, $p = .97$). Thus, whatever strategy was used to process the predominantly two-digit fractions used in our experiments did not appear to convey any selective advantage on one-digit fractions.

Overall, the results of Experiment 2 extend the findings of Experiment 1, showing a clear distance effect for all three number types. Once again, comparisons of fractions proved to be far more difficult than comparisons of decimals, and the distance effect was much more pronounced for fractions. Decimals showed an overall pattern very similar to that for matched three-digit integers, although RTs for decimals in Experiment 2 showed less sensitivity to digits beyond the tenth place, whereas RTs for integers were sensitive to digits beyond the analogous centuries place. In general, despite the greater conceptual similarity of fractions and decimals, it is decimals and integers that produce the most similar response patterns for magnitude comparisons.

Experiment 3

In Experiment 2, comparisons of decimals appear to be performed using something close to an "integer strategy," given the similarity between the response patterns observed for these two number types. However, one could reasonably argue that the decimals used in Experiment 2 fostered an integer strategy, as the length of the decimals was kept constant (three digits) and there were no decimals with leading zeros. The sets of decimals used in Experiment 2 were very similar to those used in Experiment 2 of Cohen (2010), who also reported evidence for use of an integer strategy. In his Experiment 3, Cohen included other types of decimal comparisons for which a direct analogy with multiplace integers fails (e.g., decimals such as .027). Cohen found that this more difficult set of decimal comparisons did increase RT, but only slightly (range of mean RTs from .8 to 1 s, still far less than the 2- to 8-s range we observed for fraction comparisons in our Experiment 2). Experiment 3 was performed to examine more directly whether the greater difficulty of fractions than decimals will still be observed when the decimals include those that do not correspond in magnitude to integers with the same number of digits.

Method

Participants. Participants were 26 undergraduates from the University of California, Los Angeles (mean age = 20 years; 23 females), who received course credit.

Design and material. As in Experiment 2, three different types of numbers were used for magnitude comparisons: integers, decimals, and fractions. Number type was manipulated within subjects (as in Experiment 1). Participants completed a block of fraction comparisons, of decimal comparisons, and of integer comparisons, in counterbalanced order. The reference values were the same as in Experiment 2: integer 613, decimal .613, and fraction 3/5. The set of targets used was derived from that used in Experiment 2 but with some changes to manipulate the length of target decimals (either two, three, or four digits), and to include decimals in which the first digit was a 0. The full list of targets is provided in Table 3. To provide variety in format relative to the earlier experiments, the decimals used in Experiment 3 did not include a 0 before the decimal point (e.g., .569 rather than 0.569).

Procedure. The procedure closely followed that of Experiment 2, except that the number type was manipulated within subjects so that every participant completed comparisons of each of the three number types. The number types were all presented in blocks following the procedure outlined in Experiment 2. The order of the blocks was counterbalanced across participants.

Table 3
Fractions, Decimals, and Integers Used As Targets in Experiment 3, Paired With Their Alphabetical Code Used In Figures 5 and 6

Code	Fraction	Decimal	Integer
a	1/20	.050	50
b	3/50	.06	60
c	1/4	.25	250
d	3/10	.3	300
e	30/91	.33	330
f	28/71	.3940	394
g	31/72	.4310	431
h	32/69	.464	464
i	1/2	.5	500
j	25/49	.51	510
k	23/44	.5230	523
l	33/62	.5320	532
m	5/9	.556	556
n	29/51	.569	569
o	24/41	.5850	585
p	22/37	.595	595
q	27/43	.628	628
r	35/54	.648	648
s	13/20	.65	650
t	2/3	.6670	667
u	36/53	.679	679
v	17/25	.68	680
w	7/10	.7	700
x	5/7	.7140	714
y	3/4	.750	750
z	39/50	.78	780
aa	41/50	.820	820
ab	7/8	.8750	875
ac	49/52	.942	942
ad	46/47	.9790	979

Results

In order to assess whether there were order effects in response time and error rates, we performed a 3 (number type) × 6 (order) repeated measures ANOVA. The interaction between number type and order did not approach significance for either error rates, $F(10, 40) = .68$, $p = .74$, or response times, $F(10, 40) = .70$, $p = .63$. Accordingly, we report analyses collapsing over order of number types.

Figure 5 shows the distribution of errors as a function of the magnitudes of the target items. Participants made more errors for almost every magnitude with fractions compared with decimals and integers. Averaging across magnitudes, fractions yielded the highest error rate ($M = 25\%$, $SD = 15$) compared with decimals ($M = 5\%$, $SD = 7$) and integers ($M = 5\%$, $SD = 6$). A one-way repeated measures ANOVA showed a significant difference in errors based on number type, $F(2, 24) = 20.26$, $p < .001$. Fractions generated significantly more errors than either decimals, $t(25) = 6.37$, $p < .001$, or integers, $t(25) = 6.09$, $p < .001$.

Response times for correct decisions showed a similar pattern, as shown in Figure 6. Averaging across magnitudes, fractions yielded the slowest RTs ($M = 2.3$ s, $SD = 1.6$) compared with decimals ($M = .88$ s, $SD = .3$) and integers ($M = .73$ s, $SD = .2$) A one-way repeated measures ANOVA showed a significant difference in RT based on number type, $F(2, 24) = 13.85$, $p < .001$. Each of the number types differed reliably in their RTs: fractions versus decimals, $t(25) = 5.14$, $p < .001$; fractions versus integers, $t(25) = 5.35$, $p < .001$; and decimals versus integers, $t(25) = 2.46$, $p = .02$. However, the difference in RTs between decimals and integers (about .15 s) was considerably smaller than the difference between fractions and either decimals (1.42 s) or integers (1.57 s). The modest increase in response time for decimals compared with integers is comparable with that observed by Cohen (2010) in his Experiment 3, which also included decimals of unequal lengths.

As in our previous experiments, we performed analyses to determine whether "simpler" fractions were computed significantly faster than the more difficult fractions. Although comparisons with 1/2 tended to be considerably faster than those with 32/69 (1.47 s vs. 2.46 s; $t[28] = 1.23$, $p = .21$) or 25/49 (1.47 s vs. 2.24 s; $t[30] = 1.58$, $p = .12$), neither of these differences was reliable. Two-thirds (2/3), also a reasonably familiar fraction, was not significantly faster than 35/54 (2.28 s vs. 2.60 s; $t[33] = .54$, $p = .59$) or 36/53 (2.28 s vs. 3.15 s; $t[34] = 1.43$, $p = .16$). Thus, we did not find clear evidence that "simpler" fractions were processed in a different way than other fractions, although the trends suggest that some participants may have done so.

Figure 6 shows a strong distance effect for fractions and a smaller distance effect for decimals and integers (RT data rescaled in Figure 6, right). As in the previous experiments, we conducted a linear regression with log Dist as the predictor. This predictor was reliable for fractions ($\beta = -.71$, $t[28] = 5.33$, $p < .001$) with 49% of the variance accounted for, decimals ($\beta = -.36$, $t[28] = .36$, $p = .048$) with 10% of the variance accounted for, and integers ($\beta = -.54$, $t[28] = 3.36$, $p = .002$) with 26% of the variance accounted for. For decimals, when distance was computed using values truncated to the tenth place, log Dist accounted for significantly more variance, 37% ($\beta = -.63$, $t[28] = 4.28$, $p < .001$). Indeed, log Dist based on the entire decimal did not add significant variance after accounting for the effect of the tenth

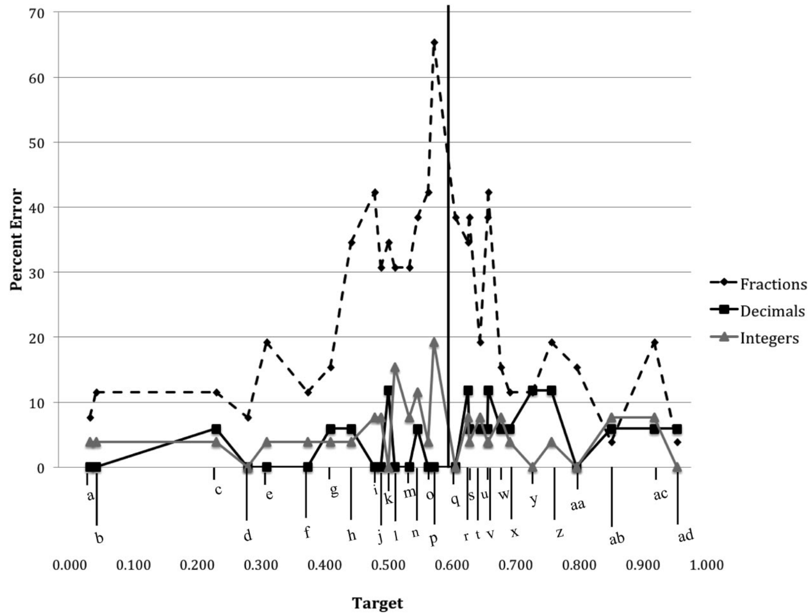

Figure 5. Distribution of percent errors across target magnitudes for fractions, decimals, and integers (Experiment 3). The solid vertical line marks the reference value. See Table 3 for identities of targets indicated by code on x-axis.

place only, $F(1, 27) = 1.63$, $p = .21$. Thus, in Experiment 3, in which the length of the decimals varied, participants apparently used a strategy of focusing on the value of the first digit. This strategy would be effective because there were no decimals with 6 in the tenths place that were lesser in magnitude than the reference value.

As in Experiments 1 and 2, we also found that the distance effect for fractions was more pronounced than for decimals or integers. Linear regressions showed that log Dist significantly predicted the *difference* in RT between the fractions and decimals ($\beta = -.74$, $t[28] = 5.79$, $p < .001$; adj $R^2 = 53\%$) and fractions and integers ($\beta = -.70$, $t[28] = 5.18$, $p < .001$; adj $R^2 = 47\%$), but not between decimals and integers ($\beta = -.04$, $t[28] = .21$, $p = .84$; adj. $R^2 = 3\%$). In addition, intercorrelations between decimals and integers were significantly related, $r(28) = .44$, $p = .015$, but neither fraction and decimal RTs, $r(28) = .28$, $p = .13$, nor fraction and integer RTs, $r(28) = .29$, $p = .12$, were significantly related.

In order to further evaluate the effect of the length of decimals, we also compared response times for decimals close in magnitude that varied in length. We found no significant difference in re-

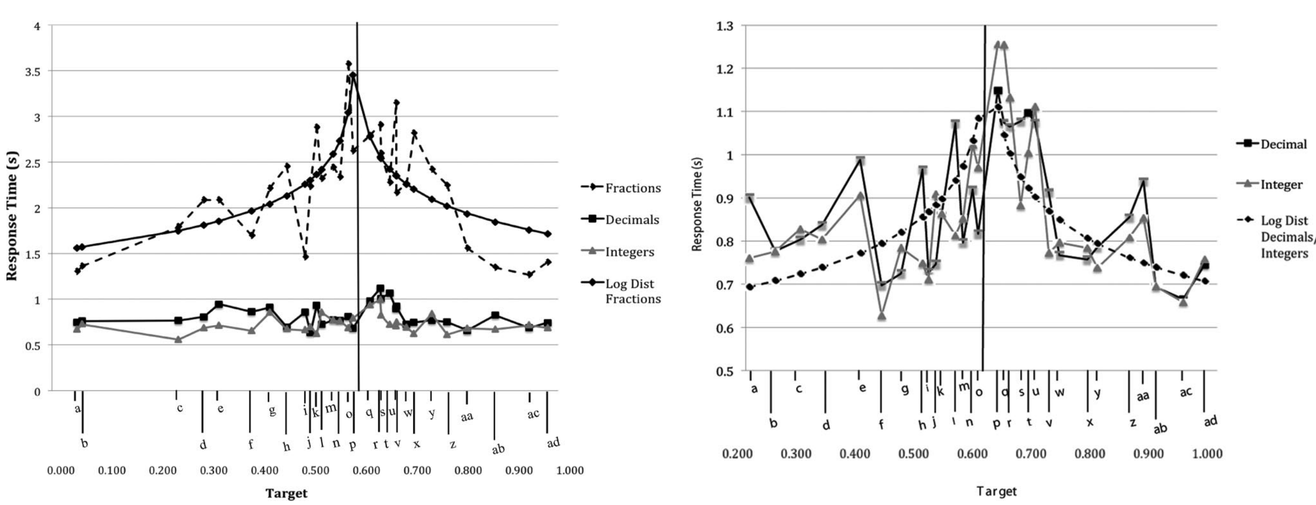

Figure 6. Distribution of mean response times across target magnitudes for fractions, decimals, and integers (Experiment 3). The solid vertical line marks the reference value. See Table 3 for identities of targets indicated by code on x-axis.

sponse times in these pairs. For example, the RT for .25 was not significantly different from .3 (.77 s vs. .81 s), $t(49) = .40$, $p = .69$. Also, the RT for .65 was not significantly different from that for .6670 (1.01 s vs. 1.07 s), $t(47) = .40$, $p = .69$. These findings lend further support to the hypothesis that participants used the first digit to make comparisons with decimals.

An additional multiple regression was performed with the integer RTs using values truncated to the hundreds place. The truncated predictor alone accounted for 23% of the variance ($\beta = -.51$; $t[28] = 3.09$, $p = .004$), and log Dist based on the entire number did not significantly increase the variance accounted for, $F(1, 27) = 2.09$, $p = .16$. Thus, the pattern of response times suggested that participants used componential strategies when comparing either decimals or multidigit integers.

Overall, the results of Experiment 3 indicate that even when participants cannot use a direct integer strategy when making comparisons with decimals, they are still considerably faster and more accurate in comparing decimals than fractions. Performance on decimal comparisons was again most similar to integer comparisons, whereas comparisons with fractions were distinctly more difficult than comparisons with either decimals or integers.

General Discussion

The central goal of the present study was to assess the similarities and differences among the processes and representations used to compare magnitudes with different types of rational numbers, especially fractions and decimals. Although conceptually similar, fractions and decimals have very different formal structures: Fractions have a bipartite structure (numerator and denominator), whereas decimals are formed using a base-ten place-value system, as are multidigit integers. Thus, fractions have a distinct componential structure; decimals and multidigit integers share a different componential structure, based on place values.

Across three experiments, we found that magnitude comparisons with all three number types yielded a distance effect, such that response times were a decreasing function of the logarithm of the numerical distance between the target and reference values. However, comparisons of fractions stood out as by far the most difficult of the three number types, and also the most sensitive to distance. Fractions were much more difficult to compare than decimals, even in Experiment 3, in which the decimal format included targets with different numbers of digits (e.g., .3, .3940) and targets with 0 as the first and/or last digit (e.g., .050). The extra difficulty of fractions appeared to be ubiquitous, even for "simple" fractions (e.g., 1/2) when embedded within a set dominated by fractions in more complex forms. In addition, across all three experiments, higher correlations were obtained between performance on decimals and integers than between performance with either of these with fractions. Together, these findings suggest that adults process decimals and integers highly similarly, whereas fractions are processed in different ways.

Both the qualitative and quantitative pattern of RTs for fraction comparisons observed in the present study RTs closely resembles that reported by Schneider and Siegler (2010). Schneider and Siegler found that RTs for fraction comparisons ranged from about 2 to 15 s (their Experiment 2), whereas RTs in the present study ranged from range from about 2 to 8 s (for our Experiment 2, which most closely matched Schneider and Siegler's procedures). The present study not only confirms the substantial absolute difficulty of comparisons with fractions but also provides a direct comparison with the much lower RTs we observed for comparisons of decimals and integers.

The greater impact of numerical distance on time to compare fractions suggests that the magnitude representations associated with fractions are less precise than those for the other number types, as would be expected if magnitudes for fractions are generated by strategies for approximation. Evidence for a less precise magnitude representation for fractions has been found in other studies. Kallai and Tzelgov (2009) found that measures of automatic processing, such as the size congruity effect, are not observed in comparisons of nonunit fractions. Kallai and Tzelgov argue that magnitude values for fractions are not discretely stored in long-term memory, but rather must be calculated online to perform arithmetic tasks (also see Iuculano & Butterworth, 2011). The present findings lend support to this interpretation.

As discussed earlier, fractions pose challenges throughout learning because of their characteristic differences from whole numbers, both conceptually and in format. Understanding how to translate the fraction in *a/b* format to a quantifiable magnitude is very difficult for children (Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004). Moreover, the present study establishes that generating magnitudes of fractions is also difficult for adults. Fractions remain more difficult than decimals, even though the former number type is introduced earlier in school. It is noteworthy that in many research articles (including the present one) that deal with fraction magnitudes, the authors often translate fractions into their decimal equivalents when trying to convey the magnitudes to the reader (e.g., Schneider & Siegler, 2010, display their results on graphs for which the x-axis is labeled not with the actual fractions but with their decimal equivalents).

The greater ease of comparing decimals than fractions, coupled with the overall similarity of decimal and integer comparisons, strongly suggests that the formal similarity of decimals and integers underlies the relative ease of processing the latter number types. Even when the deviations between the formal structures of decimals and integers are taken into consideration (as in Experiment 3), the processing difficulty of decimals is much more similar to that of matched integers than matched fractions. Of course, the fact that the format of decimals permits (and indeed encourages) use of a place-value strategy (similar to that available for multidigit integers) may well be the key to their computational advantage over fractions. The consistent compositional structure of decimals and integers affords all the advantages of metric measures over traditional imperial measures.

At the same time, detailed analyses of response times for decimals and multidigit integers suggest that participants often used componential strategies to make comparisons for these number types, focusing primarily on the first digit (the tenths value for decimals, the century value for three-digit integers). Only Experiment 1, in which specific numerical targets were repeated multiple times, yielded clear evidence that decimal comparisons were based on more than the first digit. The tendency to focus on the first digit appeared to be particularly strong in Experiment 3, in which the forms of the decimal targets were highly variable (different lengths in digits). Previous studies with both multidigit integers (Dehaene et al., 1990; Ganor-Stern et al., 2007; Nuerk et al., 2001; Vergut & De Moor, 2005) and decimals (Cohen, 2010) have also yielded evidence for componential rather than holistic processing of magnitudes. But despite the evidence suggesting componential processing, a reliable distance effect was obtained for all number types in all our experiments. Contrary to what appears to be a common assumption, the ubiquitous distance effect is not a sufficient indicator that people are basing their numerical comparisons on holistic magnitude representations retrieved from long-term memory. For componential numbers (including all three number types investigated in the present study), strategies based on estimation and componential processing may also yield a distance effect. Nonetheless, the form of the distance effect (e.g., the steeper slope observed for fractions than for decimals or integers) may provide valuable evidence concerning the precision of the magnitude values used as the basis for comparisons.

It is likely that understanding fractions conveys important conceptual benefits. Siegler et al. (2011) argue that learning about fractions is a crucial part of developing and expanding the student's concept of number. Further, Siegler et al. (2012) found that children's understanding of fractions predicts later achievement in algebra and in overall mathematics by high school. Nonetheless, the format of fractions clearly makes it more difficult to compare magnitudes of fractions than magnitudes of decimals, even though the two types of rational numbers are conceptually quite similar. The relationship between numerical formats and performance in different types of mathematical tasks remains an important area for future research.

References

Barth, H. L. A., Mont, K., Lipton, J., & Spelke, E. S. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences of the United States of America, 102,* 14116–14121. doi:10.1073/pnas.0505512102

Bonato, M., Fabbri, S., Umiltà, C., & Zorzi, M. (2007). The mental representation of numerical fractions: Real or integer? *Journal of Experimental Psychology: Human Perception and Performance, 33,* 1410–1419. doi:10.1037/0096-1523.33.6.1410

Brannon, E. M. (2002). The development of ordinal numerical knowledge in infancy. *Cognition, 83,* 223–240. doi:10.1016/S0010-0277(02)00005-7

Brannon, E. M., & Van De Walle, G. A. (2001). The development of ordinal numerical competence in young children. *Cognitive Psychology, 43,* 53–81. doi:10.1006/cogp.2001.0756

Chen, D., Lu, H., & Holyoak, K. J. (2013). *The discovery and comparison of symbolic magnitudes*. Manuscript in preparation, University of California, Los Angeles.

Cohen, D. J. (2010). Evidence for direct retrieval of relative quantity information in a quantity judgment task: Decimals, integers, and the role of physical similarity. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 36,* 1389–1398. doi:10.1037/a0020212

Dehaene, S., & Changeux, J.-P. (1993). Development of elementary numerical abilities: A neuronal model. *Journal of Cognitive Neuroscience, 5,* 390–407. doi:10.1162/jocn.1993.5.4.390

Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analog and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance, 16,* 626–641. doi:10.1037/0096-1523.16.3.626

Gallistel, C. R. (1993). A conceptual framework for the study of numerical estimation and arithmetic reasoning in animals. In S. T. Boysen & E. J. Capaldi (Eds.), *Development of numerical competence: Animal and human models* (pp. 211–223). Hillsdale, NJ: Erlbaum.

Ganor-Stern, D., Tzelgov, J., & Ellenbogen, R. (2007). Automaticity and two-digit numbers. *Journal of Experimental Psychology: Human Perception and Performance, 33*(2), 483–496. doi:10.1037/0096-1523.33.2.483

Givvin, K. B., Stigler, J. W., & Thompson, B. J. (2011). What community college developmental mathematics students understand about mathematics, Part II: The interviews. *The MathAMATYC Educator, 2,* 4–18.

Hinrichs, J. V., Yurko, D. S., & Hu, J. M. (1981). Two-digit number comparison: Use of place information. *Journal of Experimental Psychology: Human Perception and Performance, 7,* 890–901. doi:10.1037/0096-1523.7.4.890

Holyoak, K. J. (1978). Comparative judgments with numerical reference points. *Cognitive Psychology, 10,* 203–243. doi:10.1016/0010-0285(78)90014-2

Iuculano, T., & Butterworth, B. (2011). Understanding the real value of fractions and decimals. *Quarterly Journal of Experimental Psychology (2006), 64,* 2088–2098. doi:10.1080/17470218.2011.604785

Kallai, A. Y., & Tzelgov, J. (2009). A generalized fraction: An entity smaller than one on the mental number line. *Journal of Experimental Psychology: Human Perception and Performance, 35,* 1845–1864. doi:10.1037/a0016892

Marks, D. F. (1972). Relative judgment: A phenomenon and a theory. *Perception & Psychophysics, 11,* 156–160. doi:10.3758/BF03210364

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgments of numerical inequality. *Nature, 215,* 1519–1520. doi:10.1038/2151519a0

Ni, Y., & Zhou, Y. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist, 40,* 27–52. doi:10.1207/s15326985ep4001_3

Nuerk, H. C., Weger, U., Willmes, K. (2001). Decade breaks in the mental number line? Putting the tens and units back in different bins. *Cognition, 80*(1), B25–B33. doi:10.1016/S0010-0277(01)00142-1

Richland, L. E., Stigler, J. W., & Holyoak, K. J. (2012). Teaching the meaningful structure of mathematics. *Educational Psychologist, 47,* 189–203. doi:10.1080/00461520.2012.667065

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93,* 346–362. doi:10.1037/0022-0663.93.2.346

Schneider, M., & Siegler, R. S. (2010). Representations of the magnitudes of fractions. *Journal of Experimental Psychology: Human Perception and Performance, 36,* 1227–1238. doi:10.1037/a0018170

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., . . . Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science, 23,* 691–697. doi:10.1177/0956797612440101

Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. *Trends in Cognitive Sciences, 17,* 13–19. doi:10.1016/j.tics.2012.11.004

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology, 62,* 273–296. doi:10.1016/j.cogpsych.2011.03.001

Stafylidou, S., & Vosniadou, S. (2004). The development of student's understanding of the numerical value of fractions. *Learning and Instruction, 14,* 503–518. doi:10.1016/j.learninstruc.2004.06.015

Stigler, J. W., Givvin, K. B., & Thompson, B. (2010). What community college developmental mathematics students understand about mathematics. *The MathAMATYC Educator, 10,* 4–16.

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction, 14,* 453–467. doi:10.1016/j.learninstruc.2004.06.013

Verguts, T., & De Moor, W. (2005). Two-digit comparison: Decomposed, holistic, or hybrid? *Experimental Psychology, 52,* 195–200. doi:10.1027/1618-3169.52.3.195

Received January 17, 2013
Revision received March 7, 2013
Accepted April 1, 2013 ■